Murdoch

S. P. Rigaud
Oct: 17. 1835

MERCATOR's SAILING,

Applied to the True

FIGURE of the EARTH.

WITH AN

INTRODUCTION,

CONCERNING THE

Discovery and *Determination* of that FIGURE.

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TO THE

RIGHT HONOURABLE

THE

Lord *GEORGE GRAHAM*.

MY LORD,

THE Subject of this Essay being nearly related to the Profession in which you chuse to serve your Country, I have taken the Liberty to address it to your Lordship: And am proud of this Opportunity of publickly declaring with what unfeigned Respect and Gratitude I am,

MY LORD,

Your Lordship's

Most Obliged, and most

Obedient humble Servant,

PATRICK MURDOCH.

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INTRO-

INTRODUCTION,

Concerning the Magnitude and Figure of the Earth.

I.

WHOEVER propoſes any Correction of an eſtabliſhed Rule, ought, in Reaſon, to give ſome Account of the Grounds on which he proceeds. And although, with Regard to the following Eſſay, I might the more eaſily be excuſed this Trouble, that the Principle which it ſuppoſes, is now generally known and admitted; yet ſome, who are leſs converſant in the Authors who treat of theſe Matters, may not be diſpleaſed to ſee, in one diſtinct View, what has been formerly believed, or is known to be, the true Figure of the Earth.

THE firſt Men, whilſt they lived together on their native Soil, muſt have imagined the Surface of the Earth to be one immenſe Plain, diverſified by Mountains, Valleys, Beds of Rivers, and the like. Afterwards, when they began to diſperſe themſelves, and make Excurſions into diſtant Countries, the different Elevation of the Heavenly Bodies above their new Horizon, the different Length of the Days and Nights, and different Degrees of Heat and Cold, could not fail to excite their Attention and Curioſity: Yet theſe Appearances might not at firſt ſuggeſt to them any real Change of their Horizon, or that the Surface of the Eaath was *convex*. They had ob-

obſerved, no doubt, that the top of a Mountain, or of a Tower, is elevated as one approaches it; and the contrary. This might, to them, account for the Elevation or Depreſſion of the heavenly Bodies; as a ſuppoſed Difference of Diſtance from the Sun might for the Degrees of Heat and Cold. Nor can it be gathered, but from Conjecture, when, or by whom the Diſcovery of the Earth's Convexity was made.

THAT this Opinion is very ancient, cannot be doubted. *Thales* the *Mileſian*, about 600 Years before the Birth of *Chriſt*, could foretel Eclipſes *; which ſhews that the Earth's ſpherical Figure was known, not only to him, but to the Aſtronomers of preceeding Ages, from whoſe Obſervations a Theory of the celeſtial Motions had been compoſed. His Diſciple *Anaximander* even attempted to meaſure the Earth's Circumference.

THIS Knowledge *Thales* might derive either from the *Phenicians*, or from the *Egyptians*; both which Nations he had viſited. The former, whoſe Buſineſs was Trade and Navigation, could not fail to deduce it from this ſingle Obſervation, though they had made no other; That in ſailing from any high Coaſt, the lower parts of the Land are gradually hid by the Surface of the Water, till at length the whole diſappears; That, on the contrary, in making towards Land, the tops of Mountains are firſt diſcovered, and the reſt gradually as the Ship advances.

OR may we not, with ſome reaſon, imagine that the *Egyptians* had very early made the like Obſervations at home? If they did not go to Sea, the annual Return of the *Nile* brought the Sea to them. The Country was overflowed to a conſiderable depth, their Barks rode in the Streets of the Towns, and the Towns themſelves riſing from the watery Surface gave a Proſpect like that of the *Egean* Sea with its Iſlands †. Here then there wanted neither a Horizon, nor Objects proper for Obſervation; nor an acute ſpeculative People at leiſure to profit of them.

* Herodot. Lib. I. † Id. Lib. II.

How-

HOWEVER this be, as ſoon as Geometry was cultivated, and it was found that by giving the Earth a ſpherical Figure, the Appearance now mentioned, and all others in Geography were conveniently ſolved, it was reaſonably concluded that its Figure could be no other than Spherical.

THIS naturally introduced the Enquiry concerning its Dimenſion and Magnitude: An Enquiry which was very early begun, and is not quite finiſhed at preſent. For this purpoſe various Methods were thought of; all equally ſimple and accurate in the *Theory*, but difficult in the *Execution*, and ſometimes impracticable.

OF the latter ſort may be reckoned thoſe Methods, which depend on Terreſtrial Obſervations only. As when, from the top of a high Mountain, the Angle which a Line drawn to the extreme Verge of the Horizon makes with the perpendicular, is meaſured; and from this Angle with the height of the Mountain the Semidiameter of the Earth is computed. Such Methods, I ſay, are impracticable, or at leaſt inſignificant. Becauſe it is difficult to find exactly the perpendicular Height or the Diſtances of Mountains, and no leſs ſo to meaſure the Angle that is wanted, even with the beſt Inſtrument; not to mention that the Altitude of the higheſt Mountain bears ſo ſmall a Proportion to the Earth's Semidiameter, that any Error in meaſuring the one, is greatly multiplied in determining the other.

IN the Methods which have been uſed with better Succeſs, the Diſtance of two Places on the ſame Meridian is ſuppoſed known, and likewiſe the different meridian Altitudes of a fixt Star at both places; and from a Compariſon of the given Diſtance with the Difference of Altitude, the Proportion of that Diſtance to the Circumference of the Earth is found. The Sun may be uſed inſtead of a fixt Star, if his meridian Altitude is taken at both Places on the ſame Day.—But this will be beſt explain'd by the help of *Fig.* I.

IN which CABN repreſents a meridian Circle of the Earth, that is a Circle wherein a Plane drawn through the Centre and the Poles would cut the Globe. A and B are two Places in the Circumference

of this Circle, whose Distance A B is known; A D, G B D, touching the Circle in A and B, are Lines in which the Plane of the Meridian cuts the Horizons of the two Places; and SA, SB, are two Lines, or visual Rays from a Star to the Places A and B. It is likewise taken for granted or proven,

1. THAT the Arc AB hath the same Proportion to the whole Circumference ABNA, as the Angle ACB to four right Angles. 6 *El.* 33.

2. That ZA, a Plumb-Line at A, hangs perpendicular to the Horizon A D, as does the Plumb-Line zB to the Horizon GBD. Whence ZA, zB produced, will meet in the Centre C. 3 *El.* 19.

3. THAT the Distance of a Star is incomparably greater than the Diameter of the Earth; and that therefore the Lines SA, SB may be taken for parallel.

NOW if two Observations of the Star's meridian Altitude are made at A and B, these Altitudes will be measured by the Angles SAD, SBG; and if BE, BF are drawn parallel to AZ, AD, seeing likewise SA is parallel to SB, the Angle SAD will be equal to SBF; whence the Angle FBG will be the Difference of the Altitudes observed. But because zBG, EBF are Right-Angles, if zBF common to both is taken away, there will remain FBG equal to zBE; and BE being parallel to CAZ, the Angle zBE is equal to BCA, that is, BCA is equal to the Difference of the Altitudes observed. Say therefore, As that Difference is to four right Angles, so is the Arc AB, in any known Measure, to the whole Circumference in the same Measure.

IT had been the same thing, if instead of the Altitudes above the Horizon the Distances from the Zenith SAZ, SBz had been observed; for their Difference would have been zBE equal to BCA, as before.

To

To this Method, or others reducible to it, we owe all the Measures of the Earth's Circumference, that have been taken by Authors of any Note, from *Eratosthenes* down to the present Times. The most remarkable of which I have transcribed from *Varenius* and *de Maupertuis*, and reduced as follows.

THE Circumference of the Earth contains according to

	Stadia or Furlongs.	Roman *Miles of* 8 *Stadia.*
Eratosthenes	250000	31250
Hipparchus	275000	34375
Posidonius	240000	30000
Strabo and *Ptolemy*	180000	22500
The *Arabians*		20340 Miles.

According to

	Rhinland *Perches of* 12 *Feet.*	French *Toises or Fathoms.*	English *Statute Miles of* 5280 *Feet.*
Norwood	10701219	20679000	$25036\frac{3}{11}$
Picart	10630116	20541600	$24369\frac{78}{100}$
Musschenbroek	10625107	20531920	24858

Any of which Numbers multiply'd by the Fraction $\frac{113}{355}$ will give the Diameter in the same Measure; or divided by 360 will give the Length of a Degree.

Note. THE *Paris* Foot is to the *English* nearly as 38,355 to 36; the *Rhinland* Foot to the *English* as 105 to 102; and the ancient *Roman* Foot to that of *Paris*, as 11 to 12.*

* *Histoire de l'Academie de Scien.* Ann. . . .

In this Table it may be obferved, 1°. That the Numbers of the Ancients differ widely from one another; which has made it fufpected that their *Stadia*, or Furlongs, were not of the fame length, as indeed it would be difficult to prove that they were. *Vitruvius* and *Pliny*, with moft Authors, give 8 *Stadia* to a *Roman* Mile of 1000 Paces, or 5000 Feet. But Mr. *Caffini*, from a Paffage in *Strabo*, finds, that at leaft 9 *Stadia* were contained in the fame Mile in the South of *France* *. And the Dimenfions given by Authors of the greateft of the *Egyptian* Pyramids only encreafe the Uncertainty.

But allowing that the ancient Geographers by a *Stadium* do every where mean one determinate known Meafure, as it was moft natural they fhould, their difagreement, in the prefent Cafe, is fufficiently accounted for from the bad Inftruments and little Care they ufed in Operations of this kind. How ill provided they were of Inftruments we fee from the *Arenarius* of *Archmedes*, where we find the apparent Diameter of the Sun taken by means of a Cylinder placed at fuch a Diftance from the Eye as juft to cover his Body. And *Eratofthenes*, in his famous Obfervation, had only a hollow Hemifphere with a Gnomon erected in it, whereby to meafure the Sun's Diftance from the Zenith of *Alexandria* †. Befides, they took the Diftances of Places not from an actual Survey, but from common Eftimation; and were not very folicitous whether they lay precifely in the fame Meridian.

The Moderns have performed this Work with greater Care, and, we fee, to better purpofe. Upon the revival of Arts, large well-graduated Inftruments were made, the Diftances of Places were exactly meafured, and proper Allowances made for all Deviations, both from the Horizontal Line, and from the Meridian. At length Telefcopic Sights were applied to Inftruments, by which means Angles, whether above or in the Horizon, could be taken to a much greater precifion; and the Inftrument itfelf could be better examined and rectified. Still it was troublefome to make Surveys

* *Hiftoire de l'Academie de Scien.* Ann. . . .
† *Vid.* Fragment. poft Aratum. Edit. Oxon.

of

of a proper Length, ſuch as Mr. *Norwood's*, who meaſured the Diſtance from *London* to *York* with a Chain. To remedy this, it was found ſufficient, or even better, to meaſure with the greateſt Accuracy ſome ſmaller Diſtance as a *Baſe*, from which by means of a Series of Triangles connected with it, an Arc of the Meridian of a competent Length might be deduced.

FOR it is not difficult to conceive, that in a Figure conſiſting of any Number of Triangles, all whoſe Angles are known, and whereof every two that are contiguous have one Side common, if any one Side is given in length, the Dimenſions of the whole Figure may be found by *Trigonometry*; and its Poſition with reſpect to the Meridian from an *Aſtronomical* Obſervation. Whence, if a Meridian-Line is ſuppoſed to paſs through any Angle of the Figure, and perpendiculars to be drawn to it from the other Angles, the Intercepted Segments will be known: By which means an Arc of the Meridian of any Length may be meaſured. Only theſe things are to be noted; That 'tis convenient the Triangles be as few as poſſible; that they be continued nearly in the Direction of the Meridian, and that the Stations or angular Points, whether Spires of Churches, *&c.* or Signals erected on purpoſe, be ſo ſituated, that none of the Angles be very ſmall. But all this will be beſt underſtood from Mr. *de Maupertuis Figure of the Earth determined*; where the whole Proceſs is particularly deſcribed, and illuſtrated with proper Schemes.

II.

HITHERTO the Figure of the Earth has been conſidered as perfectly *Spherical*; to which indeed it ſo nearly approaches, that the Difference was not to be diſcovered in the Method already explained, if the Arcs meaſured were nearly in the ſame Latitude, eſpecially when no ſuch Difference was ſuppoſed or look'd after. For although *Norwood*, *Picard*, or ever ſo many good Artiſts in *France* and *England*, had, by good luck, each aſſigned the juſt Length of the Degree which he meaſured, the Differences of theſe Lengths would have been no greater than might be reaſonably

2 attributed

attributed to the Errors that are unavoidable in ſuch Operations. And accordingly, however various the Length of a Degree, as rated by different Authors, had been, there never aroſe the leaſt Suſpicion of the Earth's being other than a perfect Sphere, till Mr. *Richer's* celebrated Obſervation in 1672; " That the ſame Pendulum vi- " brates ſlower near the Equator than it does in *France*." Then indeed, the true Figure of the Earth could not be long unknown; this *Phenomenon* happening in the time of *Huygens* and *Newton*, the two Perſons beſt qualified to reaſon upon it; and, by a fate peculiar to it ſelf, ſcarce ſooner appearing than it received a juſt Solution.

BUT in order to give ſome intelligible Notion of this matter, and to ſhew the Relation there is between the Motion of a Pendulum and the Figure of the Earth, it is neceſſary firſt to explain what is meant by *Centrifugal Force*; a Term of principal Uſe on this Occaſion.

IT is a known Law of Motion, that a Body impelled by another moves on equably in the Direction of the Impulſe; and alſo if the Impulſes be two, three, or any finite Number, the Body upon which they are exerted, will ſtill move in a ſtrait Line, with a Velocity and Direction reſulting from the Quantities and Directions of all the ſeveral Impulſes.

WHENCE it follows, that when a Body moves in a Circle, or other Curve-Line, it is retained in that Curve by ſome force acting upon it *continually*. Such is the Force which the Hand exerts upon a Stone whirled round in a Sling; and ſuch is that conceived to be, by which the Heavenly Bodies are retained in their Orbits.

ACTION being ever equal and contrary to *Reaction*, if we conſider the *Tenſion* of the Sling as produced by a Force acting towards the Hand as a Centre, then is that Force called *Centripetal*: But if we conſider it as a Force in the Stone, acting in a contrary Direction, it is called *Centrifugal*. And in all Bodies moved round a Centre or Axis, ſuch a centrifugal Force really is, although its Effect is not always diſcernible.

IN

In solid Bodies revolving round an Axis, every Particle tends to recede from the Centre, but the Effect is destroyed by the stronger Cohesion of the Parts; 'tis otherwise in those that are fluid or very soft; as we may see in the circular Motion of a Bucket of Water suspended by a String, in the Motion of a Grindstone, and many other Instances that are familiar to every one.

One Law of centrifugal Force is, that in the same or equal Particles of Matter, supposing likewise the times of Revolution equal, its quantity will be proportional to the Semidiameter of the Circle described. Whence we may easily gather, that *in the Circumvolution of fluid Bodies, the total Effect, or the Sum of the centrifugal Forces is proportional to the Square of the Radius of Circumvolution.*

For if in the Right Line CB (*Fig.* II.) there be taken, CA, CB equal to the Lengths of two small Tubes containing a homogeneous Fluid, and which are supposed to revolve, in the same time, round a Centre C: And if from A and B there be drawn at right Angles the Lines AE, BD, proportional to CA, CB; then will CED be a right Line; and any Line *pt* parallel to AE, and meeting the Sides of the Triangle in *p* and *t*, will represent the centrifugal Force at the Distance *p*C. Now if this centrifugal Force is multiplied into a given Particle of the Fluid *pq*, the Rectangle *qt* will be proportional to the *vis motrix*, or *momentum* of that particle. And the Sum of all these Momenta will be as the Sum of all the little Rectangles *qt*, that is, as the Area of the Triangle CAE. In like manner, the Sum of all the *Momenta*, or centrifugal Forces in the longer Tube will be as the Area CBD; which Areas are in the duplicate Ratio of the Sides CA, CB. 6 *El.* 19.

To confirm this, the following Experiment may be made. Let a Brass Tube of the Shape that is represented in *Fig.* III. be filled with Mercury, or with some tinged Liquor, to the height of the Line BD, and then fixed to a Machine which turns horizontally with an equable Velocity, but so as the Axis of the Leg DC, may coincide with the Axis of Motion. Then will the Fluid in the one Leg sink

from

from D to E, and rise in the other from B to G; and FG, the Difference of the Heights of the Columns AG, CE, will be as the Sum of the centrifugal Forces in the Radius of Revolution CA; as is plain from *Hydrostatics*. This Difference FG may be measured on a fixt Gage contrived for that purpose, if a piece of Glass is cemented along an Opening made in the Leg AG, through which the Fluid may be seen.

THE Experiment may be varied several ways; as in *Fig*. IV. where the Tube is supposed to revolve about an Axis dividing AC unequally, in *c*. Then ought the Difference of the Heights AG, CE, to be proportional to the Difference of the Squares of A*c*, *c*C; making some allowance for the Tenacity and Adhesion of the Fluid. And there must now be adjusted another Tube *c*S, directly in the Axis of Motion, whereby the Fluid that rises in either Leg may be supplied.

TO apply this to the Subject in question; Let us, with Sir *Isaac Newton*, imagine, that in *Fig*. V. representing the Globe of the Earth, whose Axis is P*p*, and a Diameter of the Equator Q*q*, there passes a Canal or Tube PCQA, filled with water from P to Q. If the Earth is a Sphere of homogeneous matter, equally gravitating to the Centre at equal Distances, and is likewise at rest, then the Fluids in the Tubes PC, QC will ballance each other. But if the Sphere is made to revolve equably upon its Axis P*p*, the centrifugal Forces of the Fluid in the Leg CQ, will take off some part of its Weight: Nor can the Equilibrium be restored, till so much of the Fluid in PC hath passed over into AC, as to make the whole Weight in AC, equal to the Sum of the Weight in PC, and of the centrifugal Forces. Hence it appears,

1. That the solid Earth at present is not Spherical; else the diurnal Revolution would throw the *Ocean* upon the Parts about the Equator, causing a general Inundation for a great Tract on both sides of it; leaving those towards the Poles, if not quite dry, at least with Coasts of an immense height above the Surface of the Ocean.

2.

2. THAT if the Earth was originally fluid and ſpherical, the communicating to it its motion of diurnal Rotation, muſt neceſſarily raiſe its Parts higher with reſpect to the Axis, and depreſs the Parts towards the Poles, bringing them nearer to the Centre, till it ſettled into the Figure called an *Oblate Spheroid*, generated by the Revolution of an *Ellipſe* round its leſſer Axis. Which having been demonſtrated by others, I do not at preſent inſiſt on.

NEITHER will this Figure be ſenſibly altered from that of a geometrical Spheroid, although, as Sir *Iſaac Newton* ſuppoſes, the Earth ſhould be ſomewhat denſer towards the Centre. If we conſider this redundant Matter, which makes the exceſs of Denſity, as a diſtinct Maſs, ſimilar in Figure to the Earth, and acting upon the more homogeneous Maſs towards the Surface, in a reciprocal duplicate Ratio of the Diſtances, it is plain the more diſtant Matter towards the Equator will be comparatively lighter, than if there was no ſuch Maſs, and conſequently will riſe higher. But this riſing at the Equator, on account of the Denſity at the Centre, is confined to narrow Limits; and there muſt be a regular Subſiding, towards the Poles, correſpondent to it, which will ſtill keep the Figure of the Earth nearly to that of a perfect Spheroid.

BUT to return; The Effect of centrifugal Force, with reſpect to Gravitation, is two-fold: 1°. By it the Weight of Bodies is diminiſhed *immediately*; moſt at the Equator, and in any given Latitude, in a duplicate Ratio of its *Coſine*; or more rigorouſly as the Rectangle under the *Ordinate* and *Coſine* pertaining to that Latitude. And 2°. *mediately*, as it is the Cauſe of the Earth's Spheroidal Figure.

Now 'tis evident, that the Force by which a Pendulum vibrates, is no other, than that by which Bodies are attracted downwards. And that therefore if the Power of Gravitation was increaſed, ſo as a Body, for inſtance, which a Man could formerly raiſe from the Ground, ſhould now prove too heavy for him, though he makes an equal Effort; in this caſe a Pendulum would move faſter. And the contrary, if the Power of Gravitation was diminiſhed. It is

likewise true, that the time of the Vibration of a Pendulum is longer or shorter, when the Pendulum it self is so.

WHEN therefore Mr. *Richer* being in the Island of *Cayenne*, within five Degrees of the Equator, found that his Pendulum, which he had carried from *France*, went so much slower, that, to make it keep true Time, he must have shortned it by about an eighth part of an Inch, it was a certain Indication, that in those Parts the Force of *Gravitation* is less than it is in *France*. And wherever the Experiment has been repeated with due Care, the Conclusion has been the same, "*That the Power of Gravitation decreases in going from the Poles towards the Equator.*"

IT is true, the Warmth of the Climate contributed to lengthen Mr. *Richer*'s Pendulum, but not so much as $\frac{1}{8}$ of an Inch in a Rod of little more than three Foot; as is gathered from Experiments that have been made upon the Extension of Metals by Heat. Besides, the *French Academicians*, in their late curious Experiments on this Subject, contrived to keep the Air in the same Degree of Heat at *Pello*, in Lat. 66°. 48′. as afterwards at *Paris*; which was measured in both places by the same Thermometer.

IF therefore, from the different Lengths of a Pendulum in different Latitudes, the *Diminutions of Gravity* are found; and from the total *Diminutions* those Parts are subtracted, which result *immediately* from *centrifugal Force*, what remains will be the *Diminutions* that are owing to the *spheroidal* Figure of the Earth, supposing it every where of the same *Density*: Which reduces the Question to this; "What Species of a Spheroid will, from the General Law of Attraction, produce a given Diminution of Gravity "at a given Point of its Surface?"

IF, for Instance, the Length of a Second-Pendulum, at the Pole, is 441.38 *Lines*, or twelfth parts of a *Paris* Inch, and at the Equator 439.468, the Difference being 1.912 *Lines* *; the *Diminution of*

* See Sir *Isaac Newton*'s Table.

of Gravity will be expreſſed by the Fraction $\frac{1912}{441350}$, from which ſubtracting $\frac{1}{289}$, the *immediate* Effect of *centrifugal Force*, the remainder is $\frac{1}{1148}$. But Sir *Iſaac Newton* (*Prop.* 19. *Lib.* III.) computes, That if the Semiaxis of a Spheroid which is at reſt, is to the Semidiameter of its Equator as 100 to 101, the Diminution of Gravity at the Equator will be $\frac{1}{501}$. Say therefore, As the Diminution $\frac{1}{501}$ is to the Difference $\frac{1}{100}$; ſo is $\frac{1}{1148}$ to $\frac{1}{229}$: That is, the Semidiameter of the Equator is to half the Axis as 230 to 229.

IN the ſame Propoſition it is ſhewn, That the Fraction $\frac{1}{289}$ expreſſes likewiſe that part of the Weight of the Column CA (in *Fig.* V.) which is ſuſtained by the *centrifugal Forces*: And farther, that in the Spheroid of the laſt Paragraph, whoſe leaſt and greateſt Semidiameters are as 100 to 101, the Exceſs of Weight of the Fluid in the Column CA, above the Weight of CP will be $\frac{4}{505}$ of CA. If then the Exceſs of Weight $\frac{4}{505}$ give the Difference of Semidiameters $\frac{1}{100}$; what will the Exceſs of Weight $\frac{1}{289}$ give? The Anſwer is, $\frac{1}{229}$, as before. That is, in a Spheroid, whoſe Semidiameter of the Equator is to its Semiaxis as 230 to 229, the Exceſs of Weight in the Column CA, will be equal to that Weight which the centrifugal Forces can ſuſtain; and conſequently the Fluids in the Canals CA, CP will reſt in *Equilibrio.*

UPON this Proportion of 230 to 229, Sir *Iſacc Newton* calculates his *Table* of the Lengths of a Pendulum in different Latitudes. But the Differences of the *computed* Lengths coming out leſſer than what they had been found by Experience, he introduces the *Maſs of denſer Matter*, which was already mentioned: And increaſing the Difference of the Semidiameters in the ſame Ratio as the *obſerved* Difference of Length of the Pendulum exceeds the *computed*, raiſes the Earth from $17\frac{1}{6}$ to $31\frac{7}{12}$ Miles higher at the Equator.

OR, which will nearly ſerve to the ſame purpoſe, we may uſe the following *Rule*; " Inſtead of $\frac{1}{289}$, take a Fraction (f) which " in either of the above *Analogies* ſhall give the ſame Anſwer." Which is equivalent to this Suppoſition, " That the Effects of the

" *denser Mass, viz.* the greater Diminution of accelerating Force, " and the Elevation at the Equator correſponding to it, are *nearly* " the ſame, as if they had been produced by a *ſwifter diurnal Ro-* " *tation.*"

THUS if, according to Monſr. *de Maupertuis*'s *Table* *, the Length of a ſecond-pendulum is at the Pole $441\frac{1}{3}$ *Lines*, and at the Equator 439.33, the Difference being 2.17; the whole Diminution of Gravity will be $\frac{217}{44133}$, from which if we take $\frac{1}{289}$, and work by the former *Analogie*, the Ratio ſought will be $\frac{138}{137}$, which is too great, as $\frac{230}{229}$ is too ſmall. But if, for $\frac{1}{289}$, we uſe the Fraction $\frac{1}{174.7}$, either *Analogie* will give the Ratio of the greateſt to the leaſt Semidiameter, as 203 to 202.

Now this *Table* of Monſr. *de Maupertuis* is it ſelf calculated upon a *Hypotheſis* which had been already given up; namely the uniform Denſity of the Earth ;and differs in effect from Sir *Iſaac*'s in this only, that the *greater obſerved* Difference of Length, between *Paris* and *Pello*, upon which the Calculus proceeds, does of conſequence affect the whole *Table*. We may therefore ſuppoſe the Fraction $\frac{217}{44133}$ is too ſmall. And if inſtead of it we take $\frac{300}{44133}$, we ſhall not probably be much wide of the Truth. For, by the beſt Obſervations in Sir *Iſaac Newton*'s Time, the Difference from the Equator to *Paris* was about 2 *Lines*; from *Paris* to *Pello*, by M. *de Maupertuis*'s own accurate Experiments, it is $\frac{6}{10}$ of a *Line*, and allowing an increaſe of $\frac{4}{10}$ more from *Pello* to the *Pole*, the Sum is three *Lines*. In which Caſe, the Fraction (f) will now be increaſed to $\frac{1}{184\frac{1}{4}}$, and the Ratio of the Semidiameters to $\frac{147}{146}$.

BUT here it muſt be obſerved, that although this *Rule*, as founded on no unreaſonable Suppoſition, may ſerve till a better is diſcovered, yet is it neither demonſtrable, nor perhaps ſufficiently exact. From the Experiments with Pendulums, we may juſtly infer an Elevation of the Equator *in general*, but not its preciſe *Quantity*; this laſt depending

* *Figure of the Earth determined.*

pending partly on the internal Conſtitution of the Earth, which is unknown. And admitting that the Figure of the Earth is to be determined from a Ballance of the centrifugal and attractive Powers; as alſo, that the Diminution of *accelerating Force* at the Equator (above what is accounted for from the *centrifugal*, and from the Attraction of a homogeneous Spheroid) is owing to the *Attraction* of a denſer interior *Maſs*; yet it will not *neceſſarily* follow, that the Equator is thereby raiſed higher than if the Earth were homogeneous. Its Elevation may indeed be either *leſſer*, *equal* or *greater*, according to the *Conditions* of the *denſer Maſs*. What probable Arguments there are, that this laſt is the Caſe in Nature, ſhall be afterwards conſidered.

At present, let us ſuppoſe a *fluid homogeneous Earth* to revolve, its greateſt and leaſt Diameters being, in this State, as 230 to 229; and that afterwards a part of the *Fluid* is converted into a *concentric ſolid Sphere*, with an attractive Force, ſuch as that of the *denſer Maſs* is ſuppoſed to be. In this caſe, it is plain the Diameter of the Equator will be contracted; for in a Column of the Fluid which lies in the Plane of the Equator, the *new* Attraction is, to a Diſtance from the Centre equal to the Semiaxis, the ſame as in the *Polar* Canal; and conſequently the Attraction of the remaining part of the Equator-Column, will increaſe the Ratio of its whole Weight to that of the Column in the Axis; while in the mean time the Diminution of accelerating Force at the Equator will be greater than before.

In place of the concentric Sphere, let us ſubſtitute a ſolid Spheroid ſimilar to the Earth; the Canals communicating with each other along its Surface. And if we put the Ratio of the Semiaxis to the Semidiameter of the Equator to be that of *Unity* to z, the Heights of the two Columns of the ambient *Fluid* being in the ſame Ratio, the Weights (produced by this *new* Attraction) of two ſmall Portions of the Fluid, proportional to the whole Columns, and ſimilarly ſituated in them, will be directly as theſe Portions, and inverſely as the Squares of their Diſtances from the Centre. And *theſe* whole Weights being in a like Ratio; that of the Column in the

the Axis will be to the other as 1 to z, and z^2 to 1, that is, as z to 1; consequently it will preponderate, and raiſe the Equator higher than in the Ratio of 230 to 229.

AND if the little Spheroid is taken of a certain intermediate Species between a Sphere and that of the homogeneous Earth, its Attraction will not alter the Ratio of the Earth's Diameters.

THE Truth of theſe Concluſions may be examined as follows. Let a right Line RG (*Fig.* V.) touching a Meridian at the Equator in the Point E, repreſent the Length of a Pendulum, that is, the actual accelerating Force of Gravitation at the Equator; and, in it, ER that part which belongs to the redundant Matter. Draw to the Centre the Line GC, and from the Point R deſcribe the Curve RFZ, having its Ordinates RE, FV in the inverſe Ratio of the Squares of their Diſtances from the Centre, CE, CV. So ſhall the Area ERFV, intercepted by the Ordinate ER at the Surface of the Earth, and any other FV parallel to it, (*i. e.* the Area $RE \times EV \times \frac{CE}{CV}$) be proportional to that part of the Weight of the Column EV, which belongs to the Attraction of the *redundant* Matter: And the Trapezium EVTG, cut off by the Ordinate FV produced, ſhall repreſent that part of the Weight of the ſame Column, which reſults from the Attraction of the uniformly denſe Earth. Imagine the like Conſtruction at the Pole, marked at the analogous Points with the ſmaller Letters r, e, g, &c. and make the Area RGTF equal to $rgtf$, ſo ſhall you arrive at an Equation, expreſſing the Ratio of the greateſt and leaſt Semidiameters, anſwering to the *Data* and *Suppoſitions* upon which you proceed.

THUS, if CV is to Cv as z to *Unity*, that is, if the redundant Matter has a Form ſimilar to the Earth, the Equation produced by the Compariſon of the Areas RGTF, $rgtf$, will be

$aqb \times z^3 - \overline{\overline{a+b} \times q + rb} \times z^2 + \overline{1-a} \times \overline{q+r \times c+b} \times z + q = 0$. In which the Semiaxis of the Earth, as alſo the accelerating Force at the

the Pole, being *Unity*, z is the Semidiameter of the Equator; and (e being the Difference of the Semiaxes of the Earth, and of the redundant *Mass*) $q = e - \frac{1}{2}e^2$, $a =$ RG the accelerating Force at the Equator, $c = 1 - \overline{a + \frac{1}{289}}$, $b = \frac{100}{501}$, $r = \frac{e}{1-e} - q$.

For which, if less Accuracy is required, we may use the Quadratic Equation,

$$aqb \times z^2 - \overline{\overline{2a+b} \times q + rb} \times z + 2q + r \times \overline{c+l} = 0.$$

In like manner, if we suppose the redundant Mass to be a Sphere, whose Radius is to the Semidiameter of the Equator as t to 1, putting $a \times \overline{1 - t^2} = k$, we shall obtain the Equation.

$$\overline{\overline{c-1} \times t^2 + bk} \times z^3 + \overline{\overline{c-1} \times t^2 + 2b - k} \times z^2 + \overline{t^2 - 3 \times \overline{c+b} + 1 - k} \times z + 1 = 0.$$

Or, for three or four decimal places, the Quadratic

$$\overline{\overline{2-c} \times t^2 - bk} \times z^2 + 2 \times \overline{\overline{k-b} - ct^2} \times z + \overline{3 - t^2} \times \overline{c+b} - 2 = 0.$$

And in all Cases, the Attraction of the redundant Mass will, at the Pole of the Earth, be to the whole Attraction, as $\frac{c - b \times \overline{z-1}}{\frac{z^2-1}{z^2} - b \times \overline{z-1}}$ to *Unity*; that is, to the Attraction of the uniformly dense Earth as $c - b \times \overline{z-1}$ to $\frac{z^2-1}{z^2} - c$.

Whence likewise the Density of this Mass, will, in any given Case, be known; for it will be to the Density of the homogeneous Earth, nearly as the attractive Forces at the Pole directly, and the Cubes of the Semidiameters *inversely*. See *Corol.* 1. *Prop.* 74. *Lib.* I. *Princip.*

From these Equations it appears, that if the Fraction $\frac{25}{4415}$ expresses the whole Diminution of Gravity at the Equator, that is, supposing a Second Pendulum is shorter than at the Pole by about $2\frac{1}{3}$ Lines, and the denser Mass to be a Sphere, whose Radius is $\frac{1}{4}$ of

of the Semidiameter of the Equator, then will the Difference of the Earth's greateſt and leaſt Semidiameters be only $\frac{1}{400}$ of the latter. If the Radius of the denſer Maſs is $\frac{1}{10}$, the ſaid Difference will be very near $\frac{1}{343}$. But although the denſer Maſs were reduced to a Point, the Semidiameter of the Equator would not exceed the Semiaxis by above $\frac{1}{333}$.

On the contrary, the Fraction $\frac{25}{4413}$ remaining, if the denſer Maſs is ſimilar to the Earth, their like Diameters being as 1 to 4, the Elevation of the Equator will be between $\frac{1}{129}$ and $\frac{1}{130}$ of the Semiaxis. And if the Semiaxis of the denſer Maſs is $\frac{1}{10}$, it will riſe to between $\frac{1}{108}$ and $\frac{1}{109}$. Let the interior Spheroid become incomparably ſmall, that is, let e be greater than any Fraction whatever; then, in the firſt Equation, r is incomparably great, which reduces it to $z = \frac{c+b}{b} = 1 + \frac{1}{99}$. Or if e is leſs than any Fraction whatever, then $q = e$, and, r vaniſhing, if we divide by q, the Equation is $abz^3 - \overline{a+b} \times z^2 + \overline{1-a} \times z + 1 = 0$; which will give z nearly equal to $1 + \frac{1}{175}$. In like manner, if the total Diminution is put equal to $\frac{30}{4413}$, the *Limits* of z will be $1 + \frac{1}{80}$, and $1 + \frac{1}{167}$.*

From which Examples we ſee, that a ſpherical *Maſs* muſt contract the Semidiameter of the Equator; and in general, what the Effect of a Spheroid of any Species would be, *viz.* that *cæteris paribus*, the more *oblate* it is, or the leſſer its Magnitude, the higher it will raiſe the Column at the Equator; and the contrary. And that conſequently, as was above aſſerted, unleſs the *Conditions* of ſuch a *Maſs* are given, the Species of the Earth's Figure cannot, by any Rule yet known, be *certainly* inferred from the Experiments with Pendulums.

I would not here be miſunderſtood, as inſinuating that ſuch Experiments are of no uſe in this Subject. On the contrary, it is by them

* In all this, the Quantity of centrifugal Force is put $= \frac{1}{289}$, and $b = \frac{100}{101}$. But after theſe Fractions have ſerved for an Approximation, they may, if the Accuracy of the Experiments requires it, be calculated a-new, and the Operations repeated.

them chiefly that it can be clear'd of the Difficulties with which it is now embaraffed. What I mean is to fhew, that we ought not to be over-hafty in concluding from them, and to point out what feems to be yet wanting before we can proceed in this enquiry. " That, befides a fufficient Number of good Obfervations in all La-" titudes we can come at, the Laws of Attraction be further ftu-" died; what they are at any Point of the Surface, not only of a " homogeneous Earth, but on the fuppofitions, either that it in-" cludes a given Mafs of denfer Matter, or is gradually more denfe " from the Surface to the Centre." By comparing thefe Experiments with the *Theory*, and with each other, we may hope to fee the Figure of the Earth better determin'd, and even its internal Conftitution in part difcover'd.

Especially if we call in the Affiftance of Obfervations drawn from fome other Source; of which we are not altogether deftitute. For as we can from the Conditions of the *Mafs*, and the Diminution of Gravity, deduce the Elevation of the Equator, in the Manner already defcribed; *vice verfâ*, if, either from the Method of the *French Academicians*, which fhall be afterwards explained, or by the Obfervation of *Lunar Eclipfes*, this Elevation can be found to any tolerable Exactnefs, the Magnitude of a fimilar Spheroidal *Mafs* is had by making e the unknown Quantity in the Equation $aqbz^3 -$ &c. which will deprefs it to the Quadratic $e^2 - 3e + \frac{s}{\frac{1}{2} \times \overline{s-t}} = 0$, in which s and t are the Co-efficients of q and r in the Equation, before it was transformed. Thus, if by whatever means z is found $= 1.01$, and $a = 1 - \frac{30}{4413}$, $1 - e$, or the Semiaxis of the concentric fimilar *Spheroid*, will be a little more than .297. That it is either fimilar, or not much different, is probable; and if the Difference is confiderable, it will difcover itfelf in the Motion of Pendulums in feveral diftant Latitudes.

Although, from what is above demonftrated, it appears that the greater Diminution of accelerating Force at the Equator, is not altogether inconfiftent with a Diminution of its Semidiameter, *viz.*

 if

if we ſuppoſe the denſer Matter to be a Figure either perfectly *Spherical*, or little differing from it; yet the ſame reaſoning furniſhes a probable *Phyſical* Argument, that this is not really the Caſe in Nature. For in one of theſe Examples, where the redundant Matter was a Sphere with the Radius $\frac{1}{4}$ of the Semidiameter of the Equator, if we compute its accelerating Force at the Pole, we ſhall find it about $\frac{38}{100}$ of the whole; and conſequently the whole Denſity of the concentric Sphere would be to that of the ambient Matter as 42 to 1. Proportions which will not, I preſume, be thought very *natural*; whereas, if the redundant Maſs is a Spheroid ſimilar to the Earth, their like Diameters being as 1 and 4, its accelerating Force at the Pole will be only $\frac{48}{1000}$, and the whole Denſity of the Spheroid to that of the ambient Matter, in little more than the Ratio of 1307 to 1000. And the like might be ſhewn upon other Suppoſitions of Magnitude; whence we may reaſonably infer, that the Figure approaches much nearer to that of a Spheroid ſimilar to the Earth, than of a Sphere.

'Tis thus at leaſt that Sir *Iſaac Newton* *, the Inventor of the *Hypotheſis*, ſeems to underſtand it. Speaking of this *Maſs*, he uſes the Words *paulo denſior*; and, which Dr. *Gregory* (*Prop.* 52. *lib.* III.) ſeems to have overlooked; he meaſures its Attraction not from the Centre, as he might have done, but from *the Matter itſelf*; on purpoſe to inſinuate that he thought it was (at leaſt nearly) ſimilar in Figure to the Earth.

If it be aſked, how ſuch a Spheroid could itſelf acquire its Figure, ſeeing the centrifugal Forces were not equal to that Effect? I anſwer, That if the ambient Fluid had been perfectly homogeneous,

* *Hæc ita ſe habent ex hypotheſi quod Terra ex uniformi materia conſtat. Nam ſe materia ad centrum* paulo *denſior ſit quam ad ſuperficiem, differentiæ Pendulorum & graduum Meridiani paulo majores erunt quam pro tabulâ præcedente, propterea quod ſi materia ad centrum redundans quâ denſitas ibi major redditur, ſubducatur & ſeorſim ſpectetur, gravitas in Terram reliquam uniformiter denſam erit reciproce ut diſtantia ponderis a centro; in materiam vero redundantem reciproce ut quadratum diſtantiæ* a materiâ illâ *quam proxime. Gravitas igitur ſub æquatore minor eſt in materiam illam redundantem quam pro computo ſuperiore; & propterea Terra ibi, propter defectum gravitatis, paulo altius aſcendet & exceſſus longitudinum pendulorum & graduum ad polos paullo majores erunt quam in præcedentibus definitum eſt.* Prop. 20. Lib. III.

neous, it might be impoſſible to account for it. But if we conſider, that in the *Chaotic* ſtate, this Fluid was undoubtedly the moſt heterogeneous that can be imagined, we ſhall, in ſhewing how the central Maſs *might* acquire the Figure of a Spheroid, find a good Argument why it *could* not well receive any other.

In that heterogeneous Fluid, one Effect of its Revolution muſt have been to diſpoſe its Particles *in equilibrio* amongſt themſelves. By which means, the more denſe muſt have taken place nearer the Equator, where ſome ſmall part of their Weight being ſuſtained by the centrifugal Force, they might be reduced to an equal relative Gravity with others nearer the Pole. From this exceſs of Denſity in the Fluid nearer the Equator, would follow a ſtronger Attraction to the central Maſs, which is at firſt ſuppoſed ſpherical, and conſequently a more eaſy and copious *Accretion* to it, in the Form of a ſolid Body. And the Equator of the central Maſs being once raiſed ſenſibly higher, the Cauſe of the greater Denſity of the Fluid there, would likewiſe be a little augmented; becauſe now the Weight of a Particle would be diminiſhed, not only by the centrifugal Force, but by its Diſtance from the Centre of the Spheroid, (by *Cor.* 2. *Prop.* 91. *Lib.* I. *Princip.*) And this Proceſs, we may ſuppoſe, went on till the Fluid was purged of all the denſer Matter, that would adhere to the central Maſs, that is, till it was interrupted by the contiguity of ſome Matter, leſs diſpoſed to quit its fluid ſtate.

Such may have been the Formation of a central Body denſer than the ſurrounding Earth, and perhaps never firmly attach'd to it. Whoſe Exiſtence ſeems neceſſary not only to the Solution of the Phenomenon in queſtion, but of another that is of much greater Conſequence. For one cannot help imagining it to be the ſame with that *Nucleus*, by which Dr. *Halley* ſo ingeniouſly explains the *Magnetic Variation* (*Phil. Tranſact.* N° 195.) If this ſhould be the caſe, and if the determining its Magnitude is of any uſe in fixing the *Law* of that *Variation*, then ſhall this Enquiry be found leſs *idly ſpeculative* than, to moſt Readers, it may at preſent appear.

III.

NOTWITHSTANDING the general Agreement that had been found between Theory and Experience to give the Earth the Form of an oblate Spheroid, the Obſtinacy of ſome foreign Mathematicians render'd it neceſſary that this matter ſhould be further inquired into, and in a different Manner than had yet been uſed.

MR. *CASSINI*, in tracing the Meridian of *France* from *Dunkirk* to *Collioure* in *Rouſſillon*, had computed that a Degree of the Meridian lengthens in going Southward, ſo as to make the Earth a *Prolate* Spheroid higher at the Poles by about 95 Miles. Sir *Iſaac* remonſtrated *, " That if ſo, Bodies muſt be lighter, and a Pendu-" lum longer at the Equator than in *France* by about half an Inch; " and that the Diameter of the Earth's Shadow from South to " North, muſt exceed its Diameter from Eaſt to Weſt by 2'. 46", " a twelfth part of the Moon's Diameter. All which was manifeſtly " contrary to Fact and Obſervation." But this could not avail with Perſons who were diſpoſed to cavil.

FRANCE was chiefly the Seat of this Diſpute; where although there are Numbers of Gentlemen too well acquainted with Sir *Iſaac Newton*'s Reaſonings, not to allow them their due Weight, and too ſincerely attached to Truth to be influenced by any Prejudice or Authority; yet ſome, overawed perhaps by the Opinion of two or three noted Mathematicians of another Country, and ſupported by the Obſervations of ſo diligent an Artiſt as Mr. *Caſſini* is allowed to be, could not be ſatisfied with any Reaſon or Experiment that had been adduced. At length the KING, from a Care of the Sciences, and at an Expence, becoming ſo great a Monarch, ſent out two Companies of Mathematicians, one to *Peru*, the other as far North as they could go; who by actually meaſuring a Degree might decide this Queſtion concerning the Figure of the Earth, as being not only a matter of Curioſity, but of ſome importance in Navigation, and other practical Arts.

FOR

* *Ibid.*

For the better understanding both the Ground and the Determination of this Dispute, the Reader is desired to recollect what hath been already explained concerning the Method of Measuring an Arc of a Meridian Circle. There he will easily perceive, that (in *Fig.* I.) if the Semidiameter CA had been less, the Arc AB answering to any given Difference of Altitude of the Star S, would have been diminished proportionally; and the contrary, if CA had been greater. Now 'tis plain, that a small Arc of the Ellipse PA*p*E (*Fig.* V.) may be considered as part of the Circumference of a Circle; and that the Semidiameter of this Circle will be least, if the small Arc is taken at A, the Vertex of the great Axis; greatest, if it is taken at P; and any where else, of an intermediate Magnitude, greater or less as it approaches *this* or *that* Extreme. If therefore, as Mr. *Cassini* formerly thought †, a Degree lengthens in going towards the Equator, the Semidiameters of the *Equicurve* Circles must increase likewise, that is, the Equator will be at P, *p*, and the Poles at A, E. But the Gentlemen who went Northward by the *French* King's Order, assure us, that just the Reverse of this is true; that the Arc of a Terrestrial Meridian at the Polar Circle, answering to one Degree in the Heavens, exceeds the like Arc in *France* by a very sensible Difference.

How safely the Observations of these Gentlemen may be relied on, appears from the above-cited Treatise of Mr. *de Maupertuis*, who was at the Head of this Commission. I shall mention some of the principal Circumstances, which render their Observations less exceptionable, than any of that kind that have been made. As,

1. In the Astronomical Part, they used a Sector which could be verified to the Difference of one Second; and with which they could observe to a Degree of Exactness altogether incredible, if we did not know that this Instrument was contrived on purpose, and the Limb divided by Mr. *Graham*.

The

† He has, I am told, of late ingenuously owned his Mistake.

2. The fixt Stars which they observed (δ and α *Draconis*) were so near their Zenith, that there can be no Suspicion of any Error from the Refraction of Light.

3. The Difference of Altitude was corrected not only by the known Equation of the Præcession of the Equinoxes, but by that new Equation discovered and most ingeniously accounted for by Mr. *Bradley*, in *Phil. Transf.* N° 406.

4. Upon repeating their Observations, with the Star α, there was a Difference only of two Seconds, from the Angle they had found by the Star δ.

5. In the *surveying* Part; the *Base* which they measured was in a Surface the fittest for their purpose that can be imagined; the Ice of a River which at that Place forms it self into a sort of Lake.

6. This Base was measured with Poles precisely of the same length, by two distinct Companies, who perfectly agreed in their Numbers.

7. Mr. de *Maupertuis*, in a Discourse read to the *Royal Academy of Sciences*, shews, that in this Operation the just Length of the Base that is measured, depends upon the degree of Exactness, with which the Difference of Altitude can be observed, compared to that with which the Horizontal Angles can be taken. And applying this Theory to his Base at the Polar Circle, he finds it was of that *just* length.

8. The Angles between the Signals were observed with a Quadrant of two Foot Radius, furnished with a Micrometer. The Signals themselves were carefully reduced to the same Horizon. And the Passages of the Sun by the Verticals of two of their Signals, from which the Position of the Polygonal Figure was determined, were observed with an Instrument consisting of a Telescope, and a Hori-

Horizontal Axis upon which it moved in a vertical Circle; the Moments of these Transits being marked upon a Clock of Mr. *Graham*. The Instrument too was Mr. *Graham*'s Contrivance.

9. THE Number and Position of the Signals answered the Conditions *Pag*. xiii. They formed a Heptagonal Figure within which the Base lay, and furnished a great many Combinations of Triangles, from which the Arc of the Meridian, severally computed, was always nearly of the same Length. They put the Case, that in each of these Triangles, they had mistaken 20″ in two of the Angles, and 40″ in the Third, and that all these Errors contributed to shorten the Arc of the Meridian. And even upon this strange Supposition, there arose a Difference only of $54\frac{1}{2}$ Toises.

IF to all this we add the known Skill and indefatigable Industry of the Persons who were employed in this Affair; and that they were six in Number, who wrote down each his Observation apart, of all which, when there happened any small Difference, the *Mean* was taken; we must own that nothing better can be expected, or scarce wished for, on this Subject.

THE Result of their Operations was, "*That the Degree of the* "*Terrestrial Meridian which cuts the Polar Circle, contains* 57437.9 "*Toises; exceeding a Degree between* Paris *and* Amiens, *as mea*-"*sured by Mr.* Picart, (*after proper Allowances for the Refraction* "*of Light, the Precession of the Equinoxes and Mr.* Bradley's *E*-"*quation*) by 512.2 *Toises*." And upon this Determination the following Tables are calculated.

I must not however omit mentioning, that, since my writing this Paper, Mr. *de Maupertuis*, upon examining with Mr. *Graham*'s Sector, the Difference of Altitude observed by Mr. *Picart*, finds it is less by a few Seconds. But it was needless for this to make any Alteration in the Tables; especially seeing if ever it is thought proper to publish them in another Form, fitted for common Use, they must, by the Rules I have given, be calculated anew, or corrected, upon the best Observations that shall have then been made.

POST-

POSTSCRIPT.

THE Series given by Mr. *de Maupertuis* for determining the Ratio of the Semidiameter of the Equator to the Semiaxis, is this,

$$E \times \overline{1 + \tfrac{3}{2} \times \overline{m^2 - 1} \times S^2 + \tfrac{3}{8} \times \overline{m^2 - 1}^2 \times S^4 + \&c.} =$$
$$F \times \overline{1 + \tfrac{3}{2} \times \overline{m^2 - 1} \times s^2 + \tfrac{3}{8} \times \overline{m^2 - 1}^2 \, s^4 + \&c.}$$

In which E and F are the Lengths of the two Degrees meaſured, S and *s* the Series of the Latitudes reſpectively, to the Radius 1; the Semiaxis *m*, and the Radius of the Equator Unity. From which, putting D for $1 - m$ he draws the Formula's $D = \frac{E - F}{3 \times \overline{ES^2 - Fs^2}}$ or $D = \frac{E - F.}{3 E \times \overline{S^2 - s^2}}$.

ERRATA.

Pag. xii. *Line* 1. *dele* 1°. *Pag*. xxii. *l. penult. draw a Line over* $c+b$. *And over* $1-t^2$ *lin*. 10. *pag*. xxiii. *Ibid*. for x^2 put z^2. *Pag*. xxiv. *in the Note*, for $\frac{500}{301}$ read $\frac{100}{301}$.

In Fig. V. *ſupply an inſcribed Circle cutting* A E *in* Q, *q*.

MER-

MERCATOR's SAILING,

Applied to the

True FIGURE of the EARTH.

THE Improvement of *Navigation* being one profeſſed Deſign of the late painful and expenſive Undertaking of the *French* Academicians, I thought it might be worth while to examine how far the *Spheroidal* Figure of the Earth, reſulting from their Obſervations made at the Polar Circle, really affects the Art of Sailing; and if the Tables and Charts now in uſe, ſhall need any ſenſible Correction.

Plain Sailing, 'tis known, has no Place in long Voyages, becauſe it does not anſwer to a Round Figure. *Great Circle Sailing* gives the ſhorteſt Diſtances of Places on a Sphere, but not their Bearings, nor can agree to any Courſe that a Ship ſteers by the Compaſs, unleſs ſhe ſails in a Meridian, or in the Equator it ſelf. And although, by firſt gaining the Latitude of the Place to which a Ship is bound, and then keeping exactly in the ſame Parallel, a Port whoſe Situation Eaſtward or Weſtward is known, might be found with great certainty; yet this Method is both tedious and indirect. The only true Sailing, to which Tables and Charts muſt be adapted,

 being

being that upon a *Rumb-Line*, which cuts every Meridian at the ſame Angle.

BUT as Sea-Charts ought to be reduced to the ſimpleſt Form poſſible, that is, ought to be projected by Strait Lines only; in ſuch a Projection, if the Degrees of the Equator and its Parallels, are made all equal to each other, thoſe of the Meridian, though they were in themſelves equal, muſt be repreſented by Lines continually increaſing from the Equator to the Pole.

THE Hiſtory of this uſeful Invention, with the ſeveral Improvements it had received, may be ſeen in the *Philoſophical Tranſactions* N° 219. where Dr. *Halley* gives the genuine and moſt expeditious Method of conſtructing *Mercator*'s (or rather Mr. *Wright*'s) *Table of Meridional Parts*; deducing it, in his Maſterly Way, from the Property of the *Logarithmic Spiral*, and ſhewing that the *Nautical Meridian Line* is a Scale of *Logarithmic Tangents of the half Complements of the Latitudes.* Mr. *Cotes* demonſtrates the ſame, by his Method of *Ratios.* (*Harm. Menſur. p.* 20, 21.)

AND Mr. *George Campbel*, in a Manuſcript which I have ſeen ſeveral Years ago, laying aſide the Logarithmic Spiral, uſes only the following LEMMA.

LEMMA. FIG. I.

Let QL ($=c$) *be a Circular Arc, whoſe Complement is* LP; *let* LS ($=s$) *be its Coſine; and* LT, PT, *Tangents at* L *and* P, *meeting in* T, *either of which* (t) *will be the Tangent of* $\frac{1}{2}$ LP. *Then I ſay, the Fluxion of the Arc* QL, *will be to the Fluxion of the Tangent* t, *as the Coſine* LS *is to the ſame Tangent.*

DEMONSTRATION.

Let LS move into the Place ls, till the Arc QL is increaſed by the Quantity Ll; at l draw the Tangent lt, interſecting PT, LT in t and x, ſo ſhall Tt be the Decrement of the Tangent PT, an-

ſwering to the Increment of the Arc L l. And from x as a Centre, through T and l, Deſcribe the Arcs Tv, lq, cutting lt, LT, in v and q. Draw alſo the Semidiameter KL, which put equal to Unity. Then will $xl : ql :: x\mathrm{T} : \mathrm{T}v = \frac{ql \times x\mathrm{T}}{xl}$. But if ls is ſuppoſed to move back into LS, the Triangle Ttv will at laſt be rectilinear, $\mathrm{L}x + xq$, that is, $2lx$ will be $=\mathrm{L}l$, or $\dot{c}$, $lq = \frac{1}{2}\dot{c}^2$ (by *Cor.* 1. *Lem.* XI. Princ.) and $x\mathrm{T} = t$. Whence $\mathrm{T}v = \frac{\frac{1}{2}\dot{c}^2 \times t}{\frac{1}{2}\dot{c}} = \dot{c} \times t$. But, by the then ſimilar Triangles LKS, Ttv, LK : LS (or $1 : s$) :: Tt : $\mathrm{T}v = s \times \mathrm{T}t$; and therefore $s \times -\dot{t} = t \times \dot{c}$, or $\dot{c} : -\dot{t} :: s : t$. *Q. E. D.*

PROBLEM I. FIG. 2.

Given ABC *the Angle of a Ship's Courſe on a Sphere, with the Difference of Latitude* Aβ, *to find* CK *the Difference of Longitude.*

SOLUTION.

LET PKZ be the Meridian, and AK the Latitude whence the Ship ſets ſail, βBL the Parallel where ſhe is arrived; Br, br, Cc the Fluxions of that Parallel, of the Ship's preſent Latitude, and of the Longitude reſpectively, and the Semidiameter of the Sphere Unity. For the Coſine of the Latitude Kβ, that is, for the Sine of the Arc LP put s, and for the Tangent of $\frac{1}{2}$LP, write t. Then will Cc be to Br, as the Semidiameter of the Sphere is to the Semidiameter of the Parallel βBL, that is $\frac{\mathrm{C}c}{\mathrm{B}r} = \frac{1}{s}$. And $\mathrm{B}r : rb ::$ $n : m$ (in a given Ratio by *Hyp.*) Therefore $\mathrm{C}c = \frac{n}{m} \times \frac{rb}{s}$. But by the *Lemma* $\frac{rb}{s} = -\frac{\dot{t}}{t}$, whence $\mathrm{C}c = \frac{n}{m} \times \frac{-\dot{t}}{t}$. But if for the Tangent of half the Complement of the Latitude AK, from which the Ship ſets out, we write T, then it is plain that when CK$=o$,

 the

the Fluent of $\frac{n}{m} \times - \frac{\dot{t}}{t}$ ought to vaniſh likewiſe. Whereas it is then $= \frac{n}{m} \times \text{Fl.} - \frac{\dot{T}}{T}$. This therefore ſubtracted, CK will be $= \frac{n}{m} \times \overline{F \frac{\dot{T}}{T} - F \frac{\dot{t}}{t}} =$ the Difference of the Hyperbolic Logarithms of T and t multiplied by $\frac{n}{m}$.

Corol. I.

If A is at α (in the Equator) then $T =$ Rad. and the Difference of Longitude α C, will be $\frac{n}{m} \times \overline{l.R - l.t}$.

Corol. II.

Suppose now the curvilinear Triangle αBC is to be protracted on a Plane (*Fig.* 3.) where a right Line CD divided into equal Parts, ſhall repreſent the Equator and its Degrees, and Perpendiculars to it (CP, QP, *&c.*) The Meridians (CBP, KAP, *&c.*) then as the Sine of αBC is to the Coſine, that is, $n : m :: \alpha C : CB = \frac{m}{n} \times \alpha C$. But α C was ſhewn $= \frac{n}{m} \times \overline{l.R - l.t}$; Wherefore $CB = \overline{l.R - l.t}$. Whence it is evident, that the Degrees of Latitude will be rightly marked on the Meridian, *if a Ship is imagined to ſail from the Equator at an Angle of* 45°; *and if the Longitudes anſwering to the ſeveral Latitudes at which ſhe arrives, are transferred on the Meridian Line*; for $l.R - l.t$ (=CB) is the Longitude, when $n = m$.

In the ſame Paper Mr. *Campbel* gives a great many curious Theorems and Rules derived from this Solution, ſome of which are intirely new: But this is all I can recollect of it, and indeed all that is neceſſary for my preſent purpoſe.

PROBLEM

PROBLEM II.

LET Fig. 2. now reprefent a *Spheroid*, generated by the Revolution of an Ellipfe round its leffer Axis PZ; the Semidiameter of whofe Equator, KE is Unity, and its Semiaxis $KP = a$. And let it be required *to divide the nautical Meridian Line of this Spheroid*, or which is the fame thing, *to find the Longitude* αC *belonging to any Latitude* $K\beta$, *in a Courfe of* 45°.

SOLUTION.

LET LS, an Ordinate to the Axis from the Parallel where the Ship arrives be called z, and the Abfciffa PS, y. Draw the Tangent LM, and put rb, or $Ll = \dot{c}$, $Cc = \dot{v}$; then the Equation to this Ellipfe being $z^2 = \frac{2ay - y^2}{a^2}$, and $y = a - a \times \sqrt{1 - z^2}$, it will very eafily appear, that the Subtangent $SM = \frac{az^2}{\sqrt{1-z^2}}$. But $\dot{c} : -\dot{z} :: \sqrt{SM^2 + z^2} : z :: \frac{\sqrt{\overline{a^2-1} \times z^4 + z^2}}{\sqrt{1-z^2}} : z :: \frac{\sqrt{1 + \overline{a^2-1} \times z^2}}{\sqrt{1-z^2}} : 1 ::$ (putting $q = 1 - a^2$) $:: \sqrt{1 - qz^2} : \sqrt{1-z^2}$; or $\dot{c} = \overline{1 - qz^2}|^{\frac{1}{2}} \times \overline{1 - z^2}|^{-\frac{1}{2}} \times -\dot{z}$. But by a like reafoning as in *Prob.* 1. $\dot{v} = \frac{\dot{c}}{z}$. If therefore, for $\dot{c}$ there be fubftituted its value juft now found, exprefs'd in a Series, and the Fluents be taken, putting likewife l for a hyperbolic Logarithm, we fhall have v ($= \alpha C$) $=$

$$
\begin{array}{llllllll}
A - lz & -\frac{1}{4}z^2 & -\frac{3}{32}z^4 & -\frac{5}{96}z^6 & -\frac{35}{1024}z^8 & -\frac{63}{2560}z^{10} & -\frac{231}{12288}z^{12} & - \&c. \\
 & +\frac{1}{4}q & +\frac{1}{16}q & +\frac{3}{96}q & +\frac{5}{256}q & +\frac{7}{512}q & +\frac{21}{2048}q & + \&c. \\
 & & +\frac{1}{32}q^2 & +\frac{1}{96}q^2 & +\frac{3}{512}q^2 & +\frac{1}{256}q^2 & +\frac{35}{12288}q^2 & + \&c. \\
 & & & +\frac{1}{96}q^3 & +\frac{1}{256}q^3 & +\frac{3}{1280}q^3 & +\frac{5}{3072}q^3 & + \&c. \\
 & & & & +\frac{5}{1024}q^4 & +\frac{1}{512}q^4 & +\frac{5}{4096}q^4 & + \&c. \\
 & & & & & +\frac{7}{2560}q^5 & +\frac{7}{6144}q^5 & + \&c. \\
 & & & & & & +\frac{7}{4096}q^6 & + \&c. \\
 & & & & & & & \&c.
\end{array}
$$

Where

Where A ſtands for the Value of the Series $-l.\,z-\overline{\tfrac{1}{4}+\tfrac{1}{4}q}\times z^{2}$ &c. when z becomes equal to the Semidiameter of the Equator, with the Signs changed.

Corol. I.

If $q=0$, i. e. if the Figure is a Sphere, the Series now wanting the Terms whoſe Coefficients q enters, will exhibit the common Diviſions of the Meridian Line, and will be (*Cor.* 2. *Prob.* 1.) $=l.\,R-l.\,t$.

If the Sign of q is changed, it will give the like Diviſions for a Prolate Spheroid.

If $q=1$, $v=l.\,R-l.\,z$ is the Equation of a logarithmic Spiral, deſcribed on the Plane of the Equator.

And, *in General*, the ſame Method may be applied to any Figure generated by the Revolution of a Curve about an Axis.

Corol. II.

Setting aſide $l.\,z$, for all the following Terms in the ſame Line write g, and for the remaining Terms of the Series, which are affected with q and its Powers, put h. For the ſame things put l R, G and H, when $z=1$; then will $v=l\,R+G-H-l\,z-g+h$. But (by *Cor.* 1.) if $t=$ the Tangent of $\frac{1}{2}$ the circular Arc whereof z is the Sine, then will $l.\,R+G-l.\,z-g=l.\,R-l.\,t$; or $l.\,z+g-l.\,t-G=0$; which added to the foregoing Equation, gives $v=l.\,R-l.\,t-\overline{H-h}$.

From theſe Corollaries a *Table of Meridional Parts* for the *Spheroid* may be calculated by the following *Rules*.

I.

I.

From the Obfervations that have been made, find the Species of the Spheroid, *i. e.* the Value of q; which, according to Monf^{r}. *de Maupertuis's* Meafures, will be nearly .022.

II.

For any affumed Latitude LMS (*Fig.* 4.) whofe Sine and Cofine, to the Radius 1, are S and Σ, find the Ordinate L S$=z$; which will always be $\frac{\Sigma}{\sqrt{1-qS^2}}$.

III.

Apply, in the circumfcribed Circle $Q\lambda\pi E$, $\lambda\sigma t$ equal and parallel to LST, and by *Prob.* 1. that is, by Dr. *Halley's* Rule, find the Meridional Parts for the Latitude $Q\lambda$, or for the Angle $\lambda m\sigma$ (which in an Oblate Spheroid will always be lefs than LMS) and from thefe Parts fubtract $H-b$, reduced to the fame Denomination of Minutes, or of tenths, &c. of Minutes.

IV.

Still I ought to fet down the Method of computing the $H-b$; for in the common way of fumming up the Coefficients that ftand over each other, and multiplying them into the refpective Powers of z, it would be quite impracticable, the numeral Coefficients of q, q^2, &c. converging fo flowly. Befides, it would be neceffary when z approaches to Unity, to ufe another Series.

I confider therefore any part of $H-b$ that is affected with the fame Power of q, as the Area of a Curve. And it happens, in the prefent Cafe, that all thefe Areas are affignable in finite Terms; and that q being fo fmall, two of them, *viz.* which belong to q and q^2 are almoft always a fufficient Approximation. To find, for Example, that Part of $H-b$ which is affected with q^3; I write down the Fractions $-\frac{1}{5}+\frac{2}{3}-\frac{1}{1}$, whofe Numerators are the Unciæ of $\overline{a+b}^{3-1}$, and the Denominators the odd Numbers in an inverted Order; the Signs being alternately $\pm$, $\mp$, as the Index of q is even or

or odd. To these Coefficients I affix $x^{\frac{5}{2}}$, $x^{\frac{3}{2}}$, $x^{\frac{1}{2}}$, (having made $x = 1 - z^2$) where the Numerators of the Indices are the Denominators of the Coefficients, and the conſtant Denominator 2. And I have $-\frac{1}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} - x^{\frac{1}{2}}$, which multiplied into $-\frac{1}{16}q^3$, the Coefficient of that Term of $\overline{1-qz^2}|^{\frac{1}{2}}$ from which the Part required was produced, gives $-\frac{1}{16}q^3 \times \overline{-\frac{1}{5}x^2 + \frac{2}{3}x - 1} \times x^{\frac{1}{2}}$ for the Area ſought. In like manner the Area belonging to q is $-\frac{1}{2}q \times \overline{-\frac{2}{1}x^{\frac{1}{2}} + 0} = \frac{2}{2}qx^{\frac{1}{2}}$; that of q^2 is $-\frac{1}{8}q^2 \times \overline{\frac{1}{3}x^{\frac{3}{2}} - \frac{1}{1}x^{\frac{1}{2}}} = -\frac{1}{8}q^2 \times \overline{\frac{1}{3}x - 1} \times x^{\frac{1}{2}}$; that of q^5, $-\frac{7}{256}q^5 \times \overline{-\frac{1}{9}x^4 + \frac{4}{7}x^3 - \frac{6}{5}x^2 + \frac{4}{3}x - 1} \times x^{\frac{1}{2}}$, &c.

N. B. When z is ſmall, if it ſhould be thought convenient to calculate h ſeparately, it may be obſerved that H is equal to the Series $\frac{1}{1 \cdot 2}q + \frac{1}{3 \cdot 4}q^2 + \frac{1}{5 \cdot 6}q^3 + \frac{1}{7 \cdot 8}q^4$ ——— $\frac{1}{\overline{2n-1} \times 2n} \times q^n = .011040692$.

Or if, in the ſame Caſe, one would calculate the meridional Parts immediately from the Series, G $(= \frac{1}{4} + \frac{3}{32} + \frac{5}{96} + \text{\&c.}) = .693147$.

Construction

Construction of TABLE III.

FOR QP (*Fig.* 4.) the Quadrant of the Elliptical Meridian write Q. And let LS ($=z$) be the Semidiameter of any Parallel of Latitude; then, from the Solution of *Prob.* II. it will easily appear, that the Arc

$$\left.\begin{array}{r} QL = Q - z - \frac{1}{6}z^3 - \frac{3}{40}z^5 - \frac{5}{112}z^7 - \frac{35}{1152}z^9 - \\ + \frac{1}{6}q + \frac{1}{20}q + \frac{3}{112}q + \frac{5}{288}q + \\ + \frac{1}{40}q^2 + \frac{1}{112}q^2 + \frac{3}{576}q^2 + \\ + \frac{1}{112}q^3 + \frac{1}{288}q^3 + \\ + \frac{1}{1152}q^4 + \end{array}\right\} \text{\&c.}$$

BUT here, for the like Reasons as in the former Solution, it will be convenient to sum this Series as follows.

I.

THE first Line, setting aside Q, is the circular Arc whereof the Sine is z, to the Radius Unity; call this Arc A.

II.

FOR the Cosine of the same Arc write y, then will the second Line (i. e. $\frac{1}{2}q \times F : z^2 \times \overline{1-z^2}|^{-\frac{1}{2}}\dot{z}$) be $\frac{1}{2}q \times \overline{\frac{1}{2}A - \frac{1}{2}zy}$ ($=\frac{1}{2}q \times B$.)

III.

THE third Line $\frac{1}{8}q^2 \times \overline{\frac{3}{4}B - \frac{1}{4}z^3y}$ ($=\frac{1}{8}q^2 \times C$.)

AND the 4th is $\frac{1}{16}q^3 \times \overline{\frac{5}{6}C - \frac{1}{6}z^5y}$, *&c.* All which is plain from Mr. *de Moivre*'s 4th Theorem of Quadratures in *Philosophical Transactions* N° 278. And thus the Arcs QL may be computed with all imaginable Ease, remembering only that the Parts expressed in z and y, are to be reduced to the same Denomination with A; which, if A is expressed in Minutes, is done by multiplying them into 3437.44675.

IV.

Q, that is the whole Elliptical Quadrant QP, is, in this Method found at once; for A is now the Quadrant of a Circle, and the Products of z, y, vanish. Whence Q=

$$\overline{A - \tfrac{1}{2}q \times \tfrac{1}{2}A + \tfrac{1}{8}q^2 \times \tfrac{3}{4}B + \tfrac{1}{16}q^3 \times \tfrac{5}{6}C + \&c.} =$$
$$A \times \overline{1 - \tfrac{1}{2}q \times \tfrac{1}{2} - \tfrac{1}{8}q^2 \times \tfrac{1}{2} \times \tfrac{3}{4} - \tfrac{1}{16}q^3 \times \tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{5}{6} + \&c.}$$

V.

Or if you would compute QL independent of Q, which is convenient when QL is small, the same Expressions will still serve; observing only, *First*, That z and y now change Places, *v. g.* The third Line is now $\frac{1}{8}q^2 \times \overline{\frac{3}{4}B - \frac{1}{4}y^3 z}$, and so of the rest. *Secondly*, That the Sum of the Parts affected by q is not to be subtracted from A, but added to it. *Thirdly*, That this last Sum is to be diminished in the Ratio of the greater Axis to the lesser.

Schol. I.

If $q = 1$, the Arc QP degenerates into the Semidiameter QK, and the Equation in N° IV becomes

$$Q = A \times \overline{1 - \tfrac{1}{2} \times \tfrac{1}{2} - \tfrac{1}{8} \times \tfrac{1}{2} \times \tfrac{3}{4} - \tfrac{1}{16} \times \tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{5}{6} - \&c.}$$

$$\text{Or } Q : A :: 1 - \overline{\tfrac{1}{4} - \tfrac{1\cdot3}{4\cdot4}\alpha + \tfrac{3\cdot5}{6.6}\beta + \tfrac{5\cdot7}{8.8}\gamma + \&c.} : 1.$$

But the Series $\frac{1}{4} + \frac{1\cdot3}{4\cdot4}\alpha + \frac{3\cdot5}{6.6}\beta + \&c.$ summed by the Scholium of the 11th Proposition of Mr. *Sterling's Meth. Differ.* is = .3633803, and 1 − .3633803 (*i. e.* 6366197) : 1 :: 1 : 1.570796 :: Q : A; that is, as the Semidiameter of a Circle is to the 4th Part of its Circumference.

Schol. II.

In calculating these Tables, I have put q, that is, the Difference of the Squares of the Semiaxes, equal to .022; which by the best Surveys

Surveys that have been made, muſt be very near the Truth. * But if there were any Survey, eſpecially near the Equator, of equal Accuracy with that of Monſ^r. *de Maupertuis*, to which his could be compared, then q might be more preciſely determined. And in order to this, his Rule (Fig. of the Earth *p.* 167.) would ſcarce be ſufficiently exact. The two ſmall Errors which his Methods involve, would then diſcover themſelves; *One*, That the Degree in the Middle of the Arc meaſured, is to be rated by the Length of the whole; *the other*, That this Degree is an Arc of a Circle. The Way of proceeding would then be, *Firſt*, To approximate to q by Monſ^r. *de Maupertuis*'s Series, (taking in, if you will, another Term of it.) And then in the Spheroid thus determined, to compute (as in the Conſtruction of Tab. III.) the Lengths of the *whole* Arcs meaſured; noting the Differences between the computed and meaſured Arcs. And having repeated the like Operation with another q, ſomewhat nearer the Truth, you might from theſe Differences find, by the *Rule of Poſition*, a q that would give the *computed* and *meaſured* Arcs of the ſame Length.

So much for the *Conſtruction*; it remains now to ſhew

* Figure of the Earth determin'd, *pag.* 162, 163. *Lib.* 3. *Prop.* 18, 19, 20. *Princ.* Newton.

The USE *of the* TABLES.

I.

IF a Ship's Courſe is all in the ſame Meridian, Queſtions concerning it are ſolved by Inſpection of Tab. III. or at moſt by Subtraction. And it is only to be noted,

Firſt, THAT the Total Difference between two Arcs of the Sphere and Spheroid, reckoned from the Equator to the ſame Latitude, is greateſt at near 55° ; becauſe there the Radius of Curvature is equal to the Semidiameter of the Equator. Afterwards this Difference decreaſes, till at the Pole it ends in ſomewhat leſs than 30 Minutes.

Secondly, THAT yet the Ratio of this Difference to the whole Courſe, is perpetually decreaſing from the Equator; where it is for the firſt Degree $\frac{12}{387}$, at 55° $\frac{9}{631}$, and at the Pole no more than $\frac{198}{33702}$.

Thirdly, Courſes, the Differences of whoſe extreme Points are equidiſtant from the greateſt Difference, are the ſame as if performed on the Sphere. As in ſailing from 45° to 65° : If one Extreme is nearer the Pole, the Courſe is lenghtned; if towards the Equator, it is ſhortned.

TAB. III. muſt likewiſe be uſed in the Reduction of the rectilinear *Rumb-Lines* on the Chart, to the *Diſtances run.*

II.

IN Eaſt or Weſt Courſes, ſay, As Rad. to the Semidiameter of the Parallel (in Tab. II.) ſo are the Minutes in the Difference of Longitude to the Diſtance run. Or if the Diſtance is given, ſay, As the Semidiameter of the Parallel to Rad. ſo is the Diſtance to the Difference of Longitude.

RE-

RESUMING the Symbols z, s, Σ, q, and putting D for the Diſtance ſailed on a Parallel of the Spheroid, and d for that ſailed on the ſame Parallel of the Sphere, the common Difference of of Longitude being L, it will be $z : \Sigma :: D : d$, and $z - \Sigma : z :: D - d : D$. But becauſe it was $z = \frac{\Sigma}{\sqrt{1 - qs^2}}$, the Ratio of $z - \Sigma$ to z will be $1 - \sqrt{1 - qs^2}$; and this when greateſt, *i. e.* when $s = 1$, is $1 - \sqrt{.978} = .01106$. So that $D - d$ can never much exceed a hundredth Part of the whole Diſtance run. In like manner it is ſhewn, that, to the ſame Diſtance ſailed on a Parallel of the *Sphere* or *Spheroid*, the Differences of Longitude cannot differ by much above $\frac{1}{100}$ of that belonging to the Spheroid.

III.

TO find how the Spheroidal Figure of the Earth affects *oblique Courſes*, from Tab. I.

LET EQ (in Fig. 5.) repreſent the Equator of the nautical Chart, or any Parallel common to the Spheroid and Sphere; KH any other Parallel of the Sphere, CG the ſame Parallel of the Spheroid, EK a Meridian, and CK the Difference of meridional Parts. Biſſect CK in M, from which raiſe MR perpendicular to it; and from C, at the Diſtance ME deſcribe an Arc interſecting MR in R. Then will R be the Centre of a Circle KCA, which paſſes through K, C, and touches EQ in A; and the Angle CAK will be greater than any other CBK made by Lines drawn from C, K to a Point B in EQ that is not the Point of Contact. That is, AE *thus determined, is the Difference of Longitude of two Places* A *and* C, *which ought to be aſſumed to make the Difference of the Angles of the Courſes on the* Spheroid *and* Sphere *the greateſt poſſible.* But becauſe CK is inconſiderable in reſpect of CE, AE may be put =EM. Thus if EQ is the Equator, and CG the Parallel of 45°, A E will be =3003, and the Angle CAK will be found to be about $30'\frac{1}{2}$; greater than it would have been upon the Suppoſal of any other Difference of Longitude EB.

To compare the Angle CAK with another (*c a k*) between the Equator and any other Parallel; from A as a Centre through C describe the Arc CN, meeting AK in N; and it is plain the Angle CAK will be proportional to $\frac{CN}{CA}$. But becaufe fo fmall an Arc may be confidered as a right Line, and becaufe AKE, ACE are not much different from half right Angles, $\frac{CN}{CA}$ will be nearly as $\frac{CK}{CE}$. And feeing $\frac{CK}{CE}$, in its greateft Magnitude at the Equator, is only $\frac{11}{971}$, *i. e.* the Ratio of the Difference of the Semidiameter of the Equator; and of the Radius of Curvature, to that Radius of Curvature it follows, that the Difference of the Angles of the Courfe, which at 45° was $30\frac{1}{3}'$, can never rife to above 38′.

In the fame Figure KN (K*n*) will reprefent the Difference of the Diftance failed, and will be to the Difference of a Courfe that lies all in the fame Meridian, nearly as the Sine of KAE (KBE) to Radius.

CONCLUSION.

From all which it appears, * that the Errors in Navigation arifing from the fuppofed fpherical Figure of the Earth, are not fo confiderable as might have been apprehended; yet, fmall as they are, I believe no good Reafon can be given why they ought not be corrected. In the mixt Sciences, one Part ought not to lofe the Accuracy it can have, becaufe the other Parts are incapable of quite fo much. However difficult it is to take an Altitude at Sea, or fteer a Ship's Courfe, yet a Sea-Quadrant would hardly be reckoned good, whofe Divifions were generally wrong by 10 or 20 Minutes, fometimes by half a Degree; nor a Chart in which the Latitudes and Bearings of Places were fo much different from what they ought to be, and whofe Length fhould perhaps be an Inch too much in proportion to its Breadth; yet the Errors in the Tables and

* See the Preface to *Maupertuis*'s Figure of the Earth determined.

Charts

Charts now uſed, are equivalent to thoſe. Nor is it a good Anſwer to this, that in the Practice of Sailing, greater Errors ſtill are unavoidable. For if a Ship's Courſe is at all regulated by Tables and Charts, whatever theſe *unavoidable Errors* are, they will happen ſometimes to be increaſed by the *whole* Error of the Chart; and the Sum of two Errors may be fatal, where one Error alone would not have been ſo.

Some

Some other CONSEQUENCES *of the Earth's* Spheroidal Figure.

I.

IN Matters purely Geographical, the Earth may be consider'd as a perfect Sphere *; for Instance, in the common Stereographic Projection of the Globe, where it is only required that the Longitudes and Latitudes of Places be justly marked, it would be an idle Refinement and a spoiling the Elegance of the Projection, to substitute Ellipses for Circles, although the Difference of the Earth's Diameters was much greater than it is.

BUT in that Orthographic Projection which is made, or supposed to be made, for the easier Calculation of the Moments of Solar Eclipses, the Case is perhaps different. Were the Theory of the Moon compleated, the Differences of these Moments (of Beginning, End, or Total Obscuration) on the Sphere and Spheroid, might be, in some Circumstances, not imperceptible.

ASTRONOMERS will likewise consider how far the spheroidal Figure of the Earth may affect the Determination of the Sun's *Parallax* from the *Transit* of *Venus* in the Year 1761.

II.

THE Parallax of the Moon, at a given Distance from the Center of the Earth, will be greatest at the Equator, least at the Pole: And in any given Latitude, as at L (Fig. 4.) if perpendicular to LC, the Radius of Curvature of the Meridian, there is drawn through the Center the right Line AB, intersecting it in C; the Moon's Parallax at that Latitude will be to her Parallax at the

* Newton *Princ. Lib.* III. *Prop.* 20.

Equator,

Equator, as LC to QK. And in Lunar Eclipſes, one Diameter of the Earth's Shadow will be diminiſhed in a Ratio, eaſily aſſignable from the Sun's Declination at the time of the Eclipſe.

III.

THERE reſults likewiſe from the Earth's ſpheroidal Figure, a ſmall *Parallax* of the *Moon*'s *Azimuth* and *Right Aſcenſion*.

IN Fig. 4. Let LG repreſent the Interſection of the *Prime Vertical* of the Place L, with the Plane of the Meridian; and XY the like Interſection of a Plane paſſing through the Centre of the Earth, and parallel to the former. This laſt we may call the *Rational* Prime *Vertical*, and the former the *Senſible*. Now, becauſe not only a Semidiameter of the Earth, but even CK the Diſtance of the *Prime Verticals*, bears ſome ſenſible Proportion to the Moon's Diſtance from the Centre K; if we ſuppoſe the Moon's apparent Place to be due Weſt, *i. e.* That her Centre is in the *Senſible* Prime Vertical, ſome ſhort Time muſt intervene before it is in the *Rational* Vertical. And by the ſame Time will the Moon's Centre have paſt the *Rational Prime Vertical* on the Eaſt, ere it arrives at the *Senſible*. But from the Species of the Spheroid, and the Latitude of the Place given, the Ratio of LC to CK is given alſo; ſay therefore, As LC to CK, ſo is the Moon's *Horizontal Parallax* to that of her *Azimuth*.

THUS if we put the Latitude of the Place 45°, and the Moon's horizontal Parallax $57'\frac{1}{2}$, or $3450''$, CL will be $=9944846$, and CK$=110610$, and the Parallax of the Azimuth $38''$.

OR if this Parallax is to be computed in *Right Aſcenſion*: Suppoſing, firſt, that the Moon has no *Declination*, ſay, As LC to QK, that is in the preſent Caſe, as 9944846 to 10000000 : : ſo is $3450'' : 3469'' =$ the Moon's horizontal Parallax at the Equator. And, as QK to KF (*i. e.* 10000000 : 156426) : : ſo $3469''$ to $54''$ the *Parallax in Right Aſcenſion* required; which is in Time $3''.6$.

If the Moon has Declination towards the elevated Pole, compute what Part of her diurnal Arc is intercepted by the two vertical Planes produced: For this will be the parallax of right Aſcenſion required.

In other Verticals, the ſame Rule is to be obſerved, only it is to be remembred, that the Diſtance of the rational and ſenſible Verticals, which is the Meaſure of the azimuthal Parallax, decreaſes as the Sine of their Inclination to the Meridian, and in the Meridian itſelf vaniſhes.

S C H O L.

If the Moon's Place could be exactly calculated and obſerved, the true Figure of the Earth might receive a new Proof from Aſtronomical Obſervation. And much more eaſily might the Opinion of Mr. *Caſſini* and his Friends be confuted. For as they imagine the Earth to riſe more than twice as high at the Poles, as it really does at the Equator, we may ſuppoſe in groſs (for I have not calculated it) that this would give a *Parallax* about double to that which we have determined; and which might ſometimes riſe to above $2'$ in right Aſcenſion, or $8''$ in Time, of a contrary Denomination; becauſe the Senſible Vertical would now fall on the other ſide of the Rational.

IV.

I might add that there are, ſtrictly ſpeaking, no Antipodes, except at the Equator, and at the Poles; if there were, their erect Poſture muſt be GL: But the erect Poſture at G is G*g*, therefore, &c. The Poſture directly contrary to that at L, will be found at the other Extremity of the Diameter through L.

3.

TAB.

TAB. I. Of Meridional Parts to the Spheroid and Sphere, with their Differences.

D.	Spheroid.	Sphere.	Diff.	D.	Spheroid.	Sphere.	Diff.
1	58.7	60.0	1.3	17	1013.1	1035.3	22.2
2	117.3	120.0	2.7	18	1074.8	1098.3	23.5
3	176.1	180.1	4.0	19	1136.8	1161.6	24.8
4	234.9	240.2	5.3	20	1199.2	1225.2	26.0
5	293.8	300.4	6.6	21	1262.0	1289.2	27.2
6	352.7	360.6	7.9	22	1325.3	1353.7	28.4
7	411.8	421.0	9.2	23	1389.0	1418.6	29.6
8	471.0	481.5	10.5	24	1453.3	1484.1	30.8
9	530.4	542.2	11.8	25	1518.0	1550.0	32.0
10	589.9	603.0	13.1	26	1583.3	1616.5	33.2
11	649.7	664.1	14.4	27	1649.1	1683.5	34.4
12	709.6	725.3	15.7	28	1715.6	1751.2	35.6
13	769.8	786.8	17.0	29	1782.7	1819.5	36.8
14	830.2	848.5	18.3	30	1850.5	1888.4	37.9
15	890.9	910.5	19.6	31	1919.0	1958.0	39.0
16	951.8	972.7	20.9	32	1988.2	2028.3	40.1

D.	Spheroid.	Sphere.	Diff.
33	2058.3	2099.5	41.2
34	2129.1	2171.4	42.3
35	2200.8	2244.2	43.4
36	2273.4	2317.9	44.5
37	2347.0	2392.6	45.6
38	2421.6	2468.3	46.7
39	2497.2	2544.9	47.7
40	2573.9	2622.6	48.7
41	2651.8	2701.5	49.7
42	2730.9	2781.6	50.7
43	2811.3	2863.0	51.7
44	2893.1	2945.8	52.7
45	2976.2	3029.9	53.7
46	3060.9	3115.5	54.6
47	3147.2	3202.7	55.5
48	3235.1	3291.5	56.4
49	3324.8	3382.1	57.3
50	3416.3	3474.5	58.2
51	3509.7	3568.8	59.1

D.	Spheroid.	Sphere.	Diff.
52	3605.3	3665.2	59.9
53	3703.1	3763.8	60.7
54	3803.1	3864.6	61.5
55	3905.7	3968.0	62.3
56	4010.9	4073.9	63.0
57	4118.9	4182.6	63.7
58	4229.8	4294.2	64.4
59	4344.0	4409.1	65.1
60	4461.5	4527.3	65.8
61	4582.7	4649.2	66.5
62	4707.8	4775.0	67.2
63	4837.1	4904.9	67.8
64	4971.0	5039.4	68.4
65	5109.8	5178.8	69.0
66	5254.0	5323.6	69.6
67	5403.9	5474.0	70.1
68	5560.2	5630.8	70.6
69	5723.5	5794.6	71.1
70	5894.4	5965.9	71.5

D.	*Spheroid.*	*Sphere.*	*Diff.*
71	6073.7	6145.6	71.9
72	6262.4	6334.7	72.3
73	6461.6	6534.3	72.7
74	6672.6	6745.7	73.1
75	6896.8	6970.3	73.5
76	7136.2	7210.0	73.8
77	7393.0	7467.1	74.1
78	7670.1	7744.5	74.4
79	7970.9	8045.6	74.7
80	8300.2	8375.2	75.0

D.	*Spheroid.*	*Sphere.*	*Diff.*
81	8663.8	8739.0	75.2
82	9070.0	9145.4	75.4
83	9530.2	9605.8	75.6
84	10061.1	10136.9	75.8
85	10688.7	10764.6	75.9
86	11456.5	11532.5	76.0
87	12446.0	12522.1	76.1
88	13840.4	13916.4	76.0
89	16223.8	16299.5	75.7
90	∞	∞	37.75

TAB. II. Of the Semidiameters of the Parallels of Latitude on the Spheroid.

D.	*Semidiameter of the Parallel.*
1	9998511
2	9994043
3	9986596
4	9976174
5	9962780
6	9946413
7	9927084
8	9904791
9	9879543
10	9851345
11	9820205
12	9786130
13	9749129
14	9709209
15	9666383
16	9620661

D.	*Semidiameter of the Parallel.*
17	9572053
18	9520571
19	9466228
20	9409041
21	9349021
22	9286184
23	9220547
24	9152124
25	9080936
26	9007000
27	8930335
28	8850961
29	8768898
30	8684166
31	8596794
32	8506798

D.	*Semidiameter of the Parallel.*
33	8414206
34	8319040
35	8221327
36	8121092
37	8018364
38	7913169
39	7805539
40	7695499
41	7583084
42	7468321
43	7351245
44	7231888
45	7110282
46	6986464
47	6860468
48	6732329

D.	Semidiameter of the Parallel.
49	6602085
50	6469774
51	6335434
52	6199104
53	6060823
54	5920634
55	5778575
56	5634692
57	5489024
58	5341619
59	5192519
60	5041768
61	4889403
62	4735501

D.	Semidiameter of the Parallel.
63	4580078
64	4423193
65	4264894
66	4105227
67	3944247
68	3782000
69	3618539
70	3453915
71	3288179
72	3121383
73	2953580
74	2784825
75	2615169
76	2444669

D.	Semidiameter of the Parallel.
77	2273378
78	2101350
79	1928642
80	1755309
81	1581407
82	1406991
83	1232118
84	1056846
85	0881230
86	0705328
87	0529197
88	0352894
89	0176475
90	0

TAB.

TAB. III. Arcs of the Meridian to the Spheroid, in Minutes of the Equator.

D.	Spheroid	Sphere.	Diff.
1	58.7	60.0	1.3
2	117.3	120.0	2.7
3	176.0	180.0	4.0
4	234.7	240.0	5.3
5	293.4	300.0	6.6
6	352.1	360.0	7.9
7	410.8	420.0	9.2
8	469.6	480.0	10.4
9	528.3	540.0	11.7
10	587.0	600.0	13.0
11	645.8	660.0	14.2
12	704.5	720.0	15.5
13	763.3	780.0	16.7
14	822.1	840.0	17.9
15	880.9	900.0	19.1
16	939.7	960.0	20.3

D.	Spheroid.	Sphere.	Diff.
17	998.5	1020.0	21.5
18	1057.4	1080.0	22.6
19	1116.3	1140.0	23.7
20	1175.2	1200.0	24.8
21	1234.1	1260.0	25.9
22	1293.0	1320.0	27.0
23	1352.0	1380.0	28.0
24	1411.0	1440.0	29.0
25	1470.0	1500.0	30.0
26	1529.0	1560.0	31.0
27	1588.1	1620.0	31.9
28	1647.2	1680.0	32.8
29	1706.3	1740.0	33.7
30	1765.5	1800.0	34.5
31	1824.7	1860.0	35.3
32	1883.9	1920.0	36.1

D.	*Spheroid.*	*Sphere.*	*Diff.*
33	1943.1	1980.0	36.9
34	2002.4	2040.0	37.6
35	2061.7	2100.0	38.3
36	2121.0	2160.0	39.0
37	2180.4	2220.0	39.6
38	2239.8	2280.0	40.2
39	2299.2	2340.0	40.8
40	2358.7	2400.0	41.3
41	2418.2	2460.0	41.8
42	2477.7	2520.0	42.3
43	2537.3	2580.0	42.7
44	2596.8	2640.0	43.2
45	2656.6	2700.0	43.4
46	2716.4	2760.0	43.6
47	2776.2	2820.0	43.8
48	2835.9	2880.0	44.1
49	2895.5	2940.0	44.5
50	2955.3	3000.0	44.7
51	3015.2	3060.0	44.8

D.	*Spheroid.*	*Sphere.*	*Diff.*
52	3075.0	3120.0	44.0
53	3135.0	3180.0	45.0
54	3194.9	3240.0	45.1
55	3254.9	3300.0	45.1
56	3314.9	3360.0	45.1
57	3370.0	3420.0	45.0
58	3435.1	3480.0	44.9
59	3495.2	3540.0	44.8
60	3555.3	3600.0	44.7
61	3615.5	3660.0	44.5
62	3675.7	3720.0	44.3
63	3736.0	3780.0	44.0
64	3796.2	3840.0	43.8
65	3856.5	3900.0	43.5
66	3916.8	3960.0	43.2
67	3977.2	4020.0	42.8
68	4037.5	4080.0	42.5
69	4097.9	4140.0	42.1
70	4158.4	4200.0	41.6

D.	Spheroid.	Sphere.	Diff.
71	4218.8	4260.0	41.2
72	4279 3	4320.0	40.7
73	4339.8	4380.0	40.2
74	4400.3	4440.0	39.7
75	4460.8	4500.0	39.2
76	4521.3	4560.0	38.7
77	4581.9	4620.0	38.1
78	4642.5	4680.0	37.5
79	4703.1	4740.0	36.9
80	4763.7	4800.0	36.3

D.	Spheroid.	Sphere.	Diff.
81	4824.3	4860.0	35.7
82	4884.9	4920 0	35.1
83	4945.5	4980.0	34.5
84	5006.2	5040 0	33.8
85	5066.8	5100.0	33 2
86	5127. 5	5160.0	32.5
87	5188.2	5220.0	31.8
88	5248.8	5280.0	31.2
89	5309.5	5340.0	30.5
90	5370.2	5400.0	29.8

ALTHOUGH

ALTHOUGH the Use of these Tables in Navigation has been above explained more generally, and, 'tis hoped, to the Satisfaction of the Mathematical Reader; yet for the sake of such as are concerned only in the practical Application, I have thought fit to add what follows.

I.

Of the Construction of a Nautical Chart *to the* Spheroid of the Earth.

THIS Construction differs from *Mercator's* (which is to be found in most Books of *Navigation*) in this only; that in laying down the *Latitudes* of Places, instead of *his Table* of Meridional Parts, *Tab.* I. is to be used.

THUS, in *Fig.* 3. If A, B are two places, whereof the first is at the *Equator*, the other in the *Latitude* 12°, (North or South) the Difference of *Longitude* (AC) being 5°. 30′; from any convenient Scale of equal Parts, each of which may represent one Minute of *Longitude*, I set off AC=330 the Number of Minutes in the given Difference of Longitude; and from C having raised CP perpendicular to AC, from the same Scale, I take in it CB=709.6, the meridional Parts for the *Lat.* 12°. (above or below AC, as the Latitude is North or South.) Then having joined A, B, the Line AB will represent the *Rumb-line* between the two given Places, and the Angle ABC their *Bearing*, or the Angle of the *Course* in Sailing from the one to the other.

IF A is in the *Latitude* 16°, B in the *Latitude* 27°, (both North) the Difference of *Longitude* being 5°. 30′ as before; from 1649.1 (the meridional Parts for the greater *Latitude* 27°) I subtract 951.8 (the meridional Parts for the lesser 16°) and make CB equal to their Difference 697.3. And AC is no longer the *Equator*, but the *Parallel* of 16° North *Latitude*.

Or *lastly*, If the same things are supposed as in the foregoing Case, with this Difference, that A is on the other Side of the *Equator*; then will AC be the *Parallel* of 16° South *Latitude*, and CB will now be equal to the Sum of 1649.1, and 951.8 that is to 2601.

If, in these three Cases, the Lengths of CB had been found by *Mercator's* Tables, they would have been 725.3, 710.8, 2656.2; exceeding the former by 15.7, 13.5, 55.2 respectively.

In the same manner from the *Longitudes* and *Latitudes* given, having marked every *Port*, *Island*, *Cape*, &c. which falls within the Extent of your *Chart*, the Consequences of such Construction will be,

I.

A Line AB joining any two Places, will make with the *Meridian* an Angle ABC, equal to a Ship's Course from the one Place to the other, *i. e.* the Angle wherein the *Rumb-line* must cut every Meridian in sailing from A to B, or from B to A; which Angle may be measured Mechanically.

II.

Two Lines, parallel to the *Meridian*, drawn through the given Places A and B, and continued till they meet the (graduated) Equator, or the top or bottom of the *Chart* (where the Degrees of Longitude are likewise marked) will intercept a Distance as AC, equal to the Difference of *Longitude* of the given Places.

III.

Two Lines, through A and B, parallel to the *Equator*, and continued till they meet the *East* or *West* Margin of the *Chart*, will there intercept a Line equal to CB; and the Degrees and Minutes marked on that *Margin*, will shew the Difference of *Latitude* of the Places A and B.

IV.

IV.

THE Line AB, applied to the *Equator* as to a Scale, will not *immediately* ſhew the *true* Diſtance on a Rumb-line between A and B, but, by means of it, that Diſtance may be found. For the Line CB is not equal to the Arc of a *Meridian* between the *Parallels* of AB, but only an *Artificial* Repreſentative of it. In like manner AB is not the *natural*, but *artificial* Diſtance on the Rumb between A and B. But as AB and CB are increaſed above their *natural* Magnitude in the ſame Proportion, if a Line (L) be taken in the ſame Ratio to AB on the *Chart*, as the *natural* Length of the Arc of a Meridian, intercepted by the Parallels of A and B (to be found in *Tab.* III.) hath to the *artificial* Length CB; that Line (L) applied to the Equator, or to one of its Parallels on the Chart, will ſhew the *true* Diſtance required.

PRACTICALLY thus; In CB, take C*b* equal to the *natural* Length of the Arc of the Meridian between the Parallels of A and B; and through *b* draw *ba* parallel to BA, meeting AC in *a*, ſo ſhall *ab* be the *natural* Diſtance in the Rumb-line. 6 *El.* 4.

FROM theſe *Remarks*, the Queſtions uſually propoſed in *Mercator*'s Sailing, may be ſolved either upon a *Chart*, or by a very ſimple geometrical Conſtruction. As alſo the Reaſons of the following *Arithmetical Solutions* may be eaſily underſtood.

II.

Arithmetical Solutions of the Caſes of MERCATOR's Sailing, *on the Spheroid of the Earth.*

THE Quantities which can enter into a Queſtion of *Mercator*'s Sailing, are no more than four, *viz. The Difference of Latitude*; *the Difference of Longitude*; *the Angle of the Courſe*; and *the Diſtance ſailed.* For in the Triangle ABC (*Fig.* III.) the Angle at

C

C is a Right-Angle, and CAB is the Complement of ABC, the Angle of the Courfe. Setting afide thefe two, there remain BC, AC, ABC, and AB. Any two of which being *given*, the other two may be *required*. Whence there arife, in all, fix Cafes of *Mercator*'s Sailing; for the *Combinations* of *two* things in *four*, are *fix*. But here it is to be obferved,

I.

THAT when the *Difference of Latitude* is faid to be *given*, it is underftood that *both Latitudes* are known; and when the *Difference of Latitude* is *fought*, it is underftood that *one Latitude* is given with the Pofition of the other, becaufe otherwife it could not be known to what Part of the *Quadrant* the *Difference of Latitude* (whether *given* or *fought*) did belong; and therefore, in this Method of Solution, the Queftion would remain altogether *indeterminate*.

II.

ONE of the *fix* Cafes, juft now mentioned, is not to be folved in this Method, *viz*. When from *ab*, AC (the *Diftance run*, and *Difference of Longitude*) given, it is required to find *cb* and ABC, the *Difference of Latitude* and *Courfe*. In *Plain Sailing* the Solution would be eafy, becaufe *ab* is not there different from AB; but in *Mercator's* Sailing, feeing *cb* is unknown, its Ratio to CB will be unknown likewife; and therefore the Ratio of *ab* to AB, and the Line AB it felf. Whence the Triangle ABC cannot be conftructed nor refolved. The Cafes therefore of *Mercator*'s Sailing to be re refolved, are the five following.

CASE

CASE I.

Given the Difference of Latitude, *and* Difference of Longitude, *to find the* Course *and* Diſtance run.

EXAMPLE.

Given { N. Lat. of A. . 38° . Merid. Pts 2422
N. Lat. of B. . 5° . . Merid. Pts 294
(Whence BC=diff. . . = 2128)
AC (Diff. of Longit.)=43° = 2580

Required,
1. The Courſe ABC.
2. Diſtance (AB) *ab*.

1. { BC : AC
2128 : 2580 } :: { R : Tang. ABC.
100000 : 121241=Tang. 50°. 29′ required.

Or by Logarithms { 13.41162 = Log. 2580+Log. Rad.
3.32797 = Log. 2128
Diff. 10.08365 = Log. Tang. 50°. 29′.

2. { R : Sec. 50°. 29′.
100000 : 157158 } :: { BC : AB.
2128 : 3344 = the *Artificial* Diſt.

To reduce which to the *natural* Diſtance; from the Number of Minutes in 38°, which (*Tab.* III.) is 2240, take the Number of Minutes in 5°. which is 293, there remain 1947. Say therefore,

BC : *bc*
2128 : 1947 } :: { AB : *ab*
3344 : 3059 = the Diſtance required.

CASE

CASE II.

Given the Difference of Latitude *and* Courſe; *to find the* Difference of Longitude *and* Diſtance run.

EXAMPLE.

Given $\left\{\begin{array}{l} \text{S. Lat. A .. } 25^\circ\text{. Merid. Pts . . 1518} \\ \text{N. Lat. B .. } 30^\circ\text{. . Merid. Pts . . 1850} \\ \text{(Whence BC = Sum . . . = 3368)} \\ \text{The Angle ABC} = 43^\circ. \end{array}\right.$

Required,
1. AC.
2. (AB) *ab*.

1. $\left\{\begin{array}{l} \text{R : Tang. ABC} \\ 100000 : 93251 \end{array}\right\} : : \left\{\begin{array}{l} \text{BC : AC} \\ 3368 : 3141' = 52^\circ.\ 21' = \end{array}\right.$ Diff. of Longit. required.

By Logarithms $\left\{\begin{array}{l} 3.52737 = \text{Log. } 3368. \\ 9.96965 = \text{Log. Tang. } 43^\circ. \\ \hline 13.49702 = \text{Sum—Log. Rad.} = \text{Log. } 3141. \end{array}\right.$

2. To find (AB) *ab*, ſay, as in the former Example,

$\left\{\begin{array}{l} \text{R : Sec. } 43^\circ \\ 100000 : 136733 \end{array}\right\} : : \left\{\begin{array}{l} \text{BC : AB} \\ 3368 : 4605 \end{array}\right.$ the *Artificial* Diſt.

THEN, to 1470 (the Minutes in 25°. on the Spheroid *Tab.* III.) adding 1765, the Minutes in 30°, the Sum is 3235. Say therefore,

$\left\{\begin{array}{l} \text{BC : } bc \\ 3368 : 3235 \end{array}\right\} : : \left\{\begin{array}{l} \text{AB : } ab \\ 4605 : 4423 \end{array}\right.$ = the *natural* Diſtance required.

IF, in this Example, the Common Table of *Meridional Parts* had been uſed, the *Difference of Longitude* would have come out 3206', exceeding the truth by $1^\circ.\ 5'$; and the *Diſtance* (*ab*) would have been 4512, exceeding the *true* Diſtance by 89 Minutes, or Miles of the Equator.

CASE III.

Given the Difference of Latitude *and* Diſtance *run*; *to find the* Difference of Longitude *and Angle of the* Courſe.

EXAMPLE.

Given, as in Examp. 2.
S. Lat. A. . 25°. . Merid. Pts. 1518
N.Lat. B. . 30° . . Merid. Pts. 1850

(Whence BC = Sum = 3368
And *bc*, from *Tab.* III. = 3235)
ab - - - = 4423

Required,
1. AC.
2. Ang. ABC.

1. $\left\{\begin{matrix} bc : BC \\ 3235 : 3368 \end{matrix}\right\} :: \left\{\begin{matrix} ab : AB \\ 4423 : 4605 \end{matrix}\right.$ = the *artificial* Diſtance.

2. From the Square of AB, which is - 21206025
Take the Square of BC - - - - - - - 11343424

The Square Root of the Remainder - - 9862601, which is 3141, will be = AC the *Difference of the Longitude* ſought.

Or by the *Logarithms*; Seing $ABq - BCq = \overline{AB+BC} \times \overline{AB-BC}$, $= ACq$,

To the Log. of 7973 (= AB+BC) which is . . 3.90162
Add the Log. of 1237 (= AB—BC) which is . . 3.09237

6.99399

Half their Sum, *viz.* - - - - - 3.49699 will be the Logarithm of 3141 = AC, as before found.

3. $\left\{\begin{matrix} AB : BC \\ 4605 : 3368 \end{matrix}\right\} :: \left\{\begin{matrix} R : \text{Coſ. ABC} \\ 100000 : \text{Coſ. } 43° \end{matrix}\right.$ = Angle of the *Courſe*.

CASE IV.

Given the Difference of Longitude *and* Courſe; *to find the* Difference of Latitude *and* Diſtance ſailed.

EXAMPLE.

Given $\begin{cases} \text{N. Lat. of B} \ldots 54^\circ \\ \text{Diff. of Long. AC. } 28^\circ = 1680' \\ \text{ABC.} \ldots\ldots = 37^\circ \end{cases}$

Required,
1. (BC) *bc*
2. (AB) *ab*.

1. $\begin{Bmatrix} \text{R : Co-Tang. ABC} \\ 100000 : 132704 \end{Bmatrix} :: \begin{Bmatrix} \text{AC : BC} \\ 1680 : 2229 \end{Bmatrix}$

Now the *Meridional Parts* for the given *Latitude* of B, *viz.* 54°. are 3803. Subtract therefore the Number laſt found, 2229, from 3803, and find in *Tab.* I. what Degree and Minutes of Latitude hath the *Meridional Parts* equal to the Remainder 1574; which (taking a proportional Part) is found to be 25°. 50'. = the Latitude of A. So that the Difference of Latitude ſought, is 28°. 10'.

2. To find AB, ſay,

$\begin{Bmatrix} \text{S. ABC : R} \\ 60181 : 100000 \end{Bmatrix} :: \begin{Bmatrix} \text{AC : AB} \\ 1680 : 2791 \end{Bmatrix}$

3. In *Tab.* III. from the Minutes in 54°. *viz.* . . 3195
Take the Minutes in . . . 25°. 50'. . . . 1520

The Remainder . . . . 1675 is the quantity of the Arc *bc*. Say therefore,

$\begin{Bmatrix} \text{BC : } bc \\ 2229 : 1675 \end{Bmatrix} :: \begin{Bmatrix} \text{AB : } ab \\ 2791 : 2098 \end{Bmatrix}$ = the Diſtance ſought.

N. B.

N. B. If, in this *Example*, the Latitude of B had been 20°, then I muſt have ſubtracted from BC=2229, the Meridional Parts of 20°. *viz.* 1199, and the Remainder 1030 would have ſhewn that A was on the other ſide of the *Equator*, in the *Lat.* 17°. 16′.

CASE V.

Given the Diſtance *run, and Angle of the* Courſe; *to find the* Difference of Latitude, *and* Difference of Longitude.

EXAMPLE.

Given { N. Lat. of B . 45°; Angle ABC . . 23°; Diſtance run (*ba*) 3700′ }

Required,
1. (BC) *bc.*
2. AC.

1. $\left\{\begin{matrix} \text{R} & : \text{Coſ. ABC} \\ 100000 & : 92050 \end{matrix}\right\} :: \left\{\begin{matrix} ba & : bc \\ 3700 & : 3406 \end{matrix}\right.$ = the Arc of the Meridian intercepted by the *Parallels* through A and B.

2. In *Tab.* III. the Minutes for the given Latitude of B (45°.) are 2656. Therefore from *bc* = 3406, I take 2656; and the remainder 750, applied to the ſame *Tab.* III. ſhews that A falls in 12°. 46′. *South Latitude.* I add then the *Meridional Parts* of 12°. 46′. (*viz.* 756.) to the *Meridional Parts* of 45°. (2976.) And the Sum 3732, is the Line BC on the *Chart.*

3. $\left\{\begin{matrix} bc & : \text{BC} \\ 3406 & : 3732 \end{matrix}\right\} :: \left\{\begin{matrix} ab & : \text{AB} \\ 3700 & : 4051 \end{matrix}\right.$ = AB.

4. As in Case III. to Log. 7783 (=AB+BC) . . 3.89115
Add Log. . . . . 319 (=AB—BC) . . 2.50379

6.39494

$\frac{1}{2}$ the Sum . . . . 3.19747 is the Log. of 1576=AC sought.

THE like Operations will serve for the same Cases in common *Mercator*'s Sailing; only the Reduction of the Arcs *bc*, which is here performed by the help of *Tab.* III. is there done at once, by multiplying the Degrees in *bc* by 60; adding, if there are any, the odd Minutes.

I say nothing of *Parallel-Sailing*, having sufficiently explained it above, *Pag.* 13. nor of *Traverse* Courses, which being no more than *Combinations* of the *simple* Cases, are solved by the same Rules, though with more Labour.

I beg leave only, to subjoin the following Remarks upon the Method it self.

I.

AS I insinuated (*Pag.* 14.) the Necessity of a Correction in our common *Charts* and *Tables*, for this general Reason, that, "*In the* "*mixt Sciences, unavoidable Errors ought, by the Exactness of the* "*Theoretical Part, to be kept within the narrowest Bounds possible*;" So we see from *Examp.* II. that the Errors of *Mercator*'s Chart may sometimes considerably augment whatever others a Ship's Reckoning is subject to. And Cases may be imagined, in which the Errors would be still greater than in that Example.

II.

IF, from the Situation of two Places (A and B) with respect to *Longitude* and *Latitude*, the Angle of the *Course* is determined, as in *Case*

Caſe I. the Difference will not indeed be conſiderable, whether we uſe the *Common* Tables, or *theſe*, as never exceeding two thirds of a Degree. (*Vid. p.* 14.) But if we ſuppoſe, what ofteneſt happens, that a Ship does not ſteer her Courſe on the direct Rumb from A to B, but upon ſome other given Rumb, till ſhe is arrived at a Place (C) in a known Latitude; that in order to find her Courſe from C to the intended Port B, the *Longitude* of C is computed, as in *Caſe* II. from the *Lat.* of A, the *Lat.* of C, and the *Courſe* hitherto ſteered: And laſtly, that from the *Longitude* of C, thus computed with the *Latitudes* of C and B, her future *Courſe* is determined; it may come out very different on the *Sphere*, from what it would be on the *Spheroid.*

An Example will beſt explain this Remark.

Let the Place A be at the *Equator*; B in the N. *Lat.* 40°. Weſt of A, - - - - - -	54°. 45′.
Then, by *Caſe* I. the Angle of the Courſe on the *Spheroid* will be - - - - - -	51°. 55′.
And on the *Sphere* - - - - - -	51. 24
The Difference being no more than - - -	31

But if on the *Spheroid*, the Ship ſails NW *b* W, that is, in an Angle of 56°. 15′. till ſhe reaches C in the *Lat.* 37°. The Difference of *Longitude* of A and C, will (by *Caſe* II.) be 58°. 32′. and the Courſe from C to B will be NE, *i. e.* at an Angle of 45°. Whereas, if ſhe is ſuppoſed to ſail on a *Sphere*, the Difference of *Longitude* of A and C will be 59°. 41′. And her *Courſe* from C to B will be 52°. 9′. greater than it ought to be by 7°. 9′. *i. e.* by almoſt $\frac{3}{4}$ of a *Point*. At the ſame time the *Difference* of the *Diſtances* between C and B on the Spheroid and Sphere, will be above a ſeventh Part of the Whole.

The Reader may, for his Exerciſe, frame to himſelf other Examples, aſſuming not one intermediate Place (C), but two or three (as C, D, E:)

C, D, E:) and he will ſee how high the *Error* of the *laſt* Courſe, as it involves all the preceeding Errors of the *Longitudes*, may ſometimes riſe.

III.

In *Latitudes* under 28°, the *Plain Chart* is preferable to *Mercator*'s. From o *Lat.* to 20°. the *Meridional Diſtances* of *Mercator* exceed thoſe of the *Plain Chart*; which laſt exceed the Truth. From 20°. to 28°. the *Defect* of the *Meridional* Diſtances on the *Plain* Chart is leſs than the *Exceſs* of *Mercator*'s. After this the ſaid *Defect* increaſes quickly, ſo that, between 33°. and 34°. it is *double*, and at 38°. a little more than *triple* the *Exceſs*. Whence I infer, that for about a third part of the Quadrant, there is at leaſt as much Reaſon for correcting the *Sea-Charts* now, as formerly.

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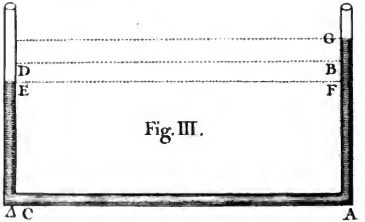
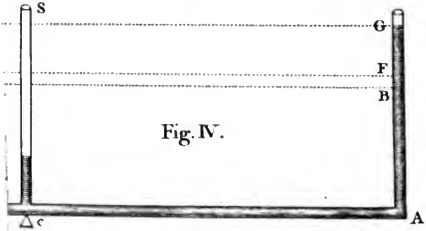


Fig. III.



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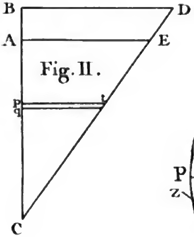

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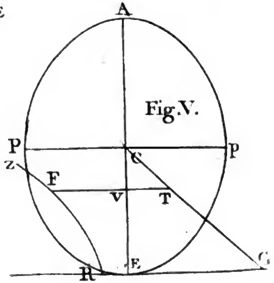

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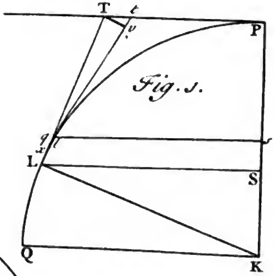

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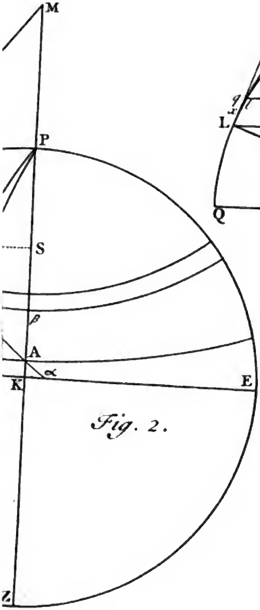

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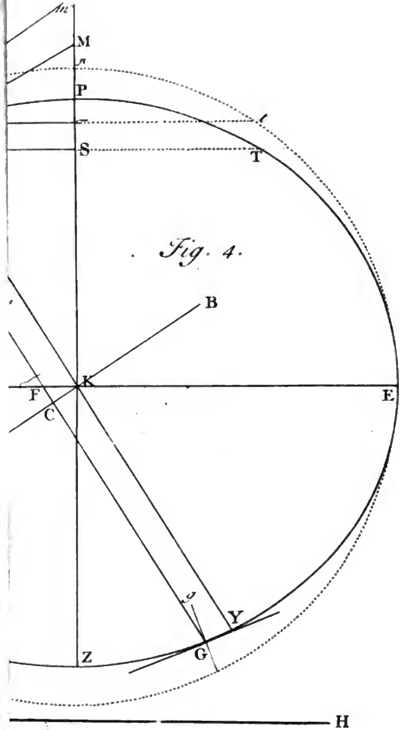

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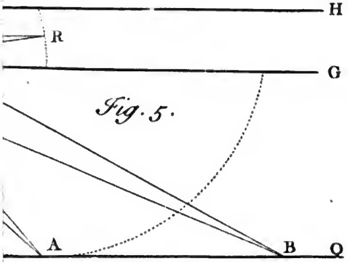

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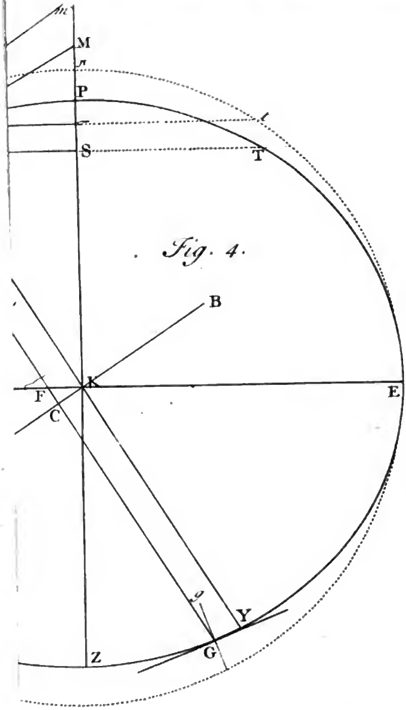

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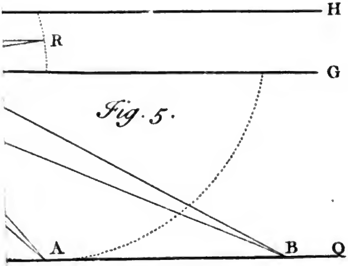

Fig. 5.